1. Define rational numbers.

The number of the form $\frac{p}{q}$ where p, q integers and $q \neq 0$ are called rational numbers. For example, $\frac{2}{9}$, 7, $\sqrt{\frac{25}{16}}$

$$Q = \left\{x | x = \frac{p}{q} \ , \quad p, q \in z \ \wedge q \neq 0\right\}$$

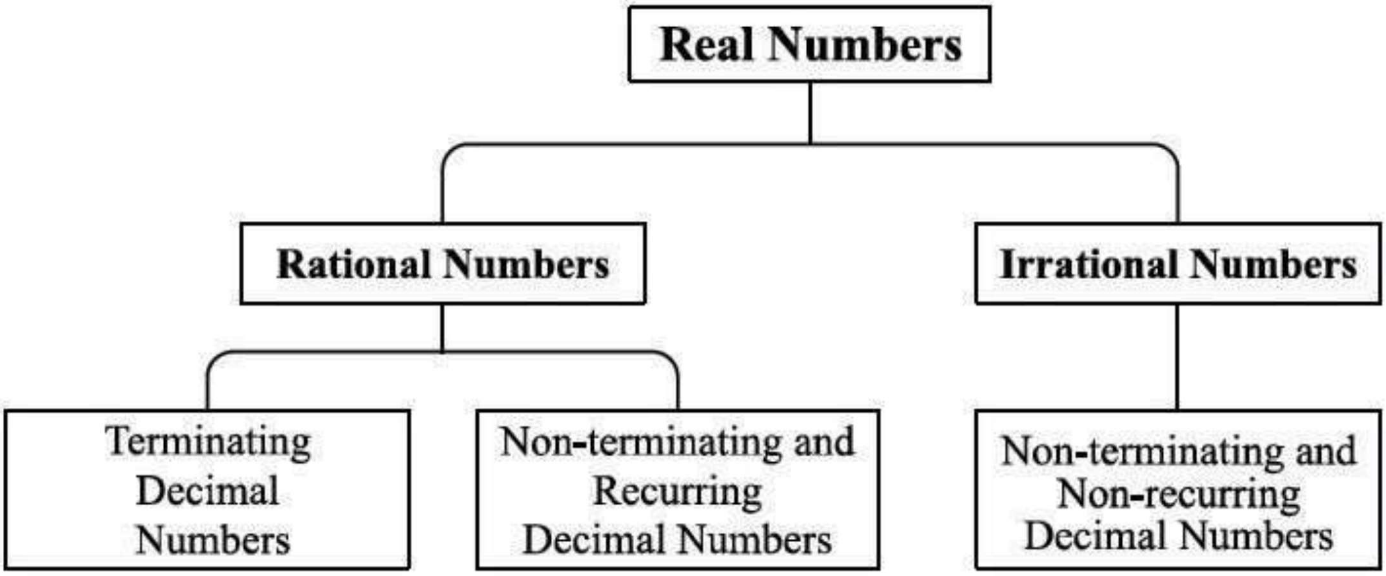
2. Define irrational numbers.

The number which cannot be express of the form $\frac{p}{q}$ where p, q integers and $q \neq 0$ are called irrational numbers. For example, $e, \pi, \sqrt{2}$

$$Q' = \left\{x | x \neq \frac{p}{q} \ , \quad p, q \in z \ \wedge q \neq 0\right\}$$

3. Define set of real numbers.

The union of the set of rational numbers and irrational numbers is known as the set of real numbers. It is denoted by $R. \ i.e. \mathbb{R} = Q \cup Q'$



4. Explain decimal representation of rational number?

The decimal representations of rational numbers are of two types.

(i) Terminating Decimal Fractions

(ii) Recurring and Non-Terminating Decimal Fractions

(i) Terminating Decimal Fractions

The decimal number with a finite number of digits after the decimal point is called a terminating decimal number. For example,

$$\frac{2}{5} = 0.4 \ and \ \frac{3}{8} = 0.375$$

(ii) Recurring and Non-Terminating Decimal Fractions

The decimal numbers with an infinitely repeating pattern of digits after the decimal point are called non-terminating and recurring decimal numbers. For example,

$$\frac{1}{3} = 0.333 \ldots = 0.\overline{3} \ \ (3 \ repeats \ infinitely)$$

$$\frac{1}{6} = 0.166 \ldots = 0.1\overline{6} \ \ (6 \ repeats \ infinitely)$$

$$\frac{22}{7} = 3.142857142857 \ldots = 3.\overline{142857} \ \ (the \ pattern \ 142857 \ repeats \ infinitely)$$

5. What are irrational numbers in terms of their decimal representation?
Decimal numbers that do not repeat a pattern of digits after the decimal point continue indefinitely without terminating.

Non-terminating and non-recurring decimal numbers are known as irrational numbers.

Examples:
- $\pi = 3.1415926535897932 \ldots$
- $e = 2.71828182845904 \ldots$
- $\sqrt{2} = 1.41421356237309 \ldots$

Note:
(i) $e = 2.7182 \ldots$ is called *Euler's Number.*
(ii) $Rational\ number\ +\ Irrational\ number\ =\ Irrational\ number.$
(iii) $Rational\ number\ (\neq\ 0)\ \times\ Irrational\ number\ =\ Irrational\ number.$
(iv) *The **product of two irrational numbers** can be either **rational** or **irrational**, depending on the numbers involved.*

6. Explain the concept of radicals and radicands.
If n is a positive integer greater than 1 and a is a real number, then any real number x such that $x^n = a$ is called (radical) the nth root of a, and in symbols is written as

$$x^n = a$$
$$[x^n]^{1/n} = [a]^{1/n}$$
$$x = a^{1/n}$$
$$x = \sqrt[n]{a}$$

In the radical $\sqrt[n]{a}$, the symbol $\sqrt{\ }$ is called the **radical sign**, n is called the **index** of the radical and the real number a under the radical sign is called the **radicand** or **base**.

7. Define surd.
An irrational radical with rational radicand is called a surd. For example,
- $\sqrt{5}$ is a surd because $\sqrt{5}$ does not give a whole number.
- $\sqrt{9}$ is **not** a surd because it simplifies to 3 (a whole number).
- $\sqrt{3}, \sqrt[3]{7}$ are surds.
- $\sqrt{\pi}$ and $\sqrt{e}$ are **not** surds.

Note: Every **surd** is an **irrational number**, but not every **irrational number** is a **surd** (e.g., $\sqrt{\pi}$ is irrational but not a surd) and the **product** of two **conjugate surds** is a **rational number**.

8. Define monomial surd.
A surd which contains a single term is called a monomial surd. For example, $\sqrt{2}, \sqrt{3}$

9. Define binomial surd.
A surd which contains sum of two monomial surds or sum of a monomial surd and a rational number is called a binomial surd. For example, $\sqrt{2} + \sqrt{7}, \sqrt{2} + 5$

10. Define conjugate surd.
Conjugate surd of $\sqrt{a} + \sqrt{b}$ is defined as$\sqrt{a} - \sqrt{b}$.

11. What are the additive properties of real numbers?

Name of the Property	∀ a, b, c ∈ ℝ	Examples
Closure	$a + b \in R$	$2 + 3 = 5 \in R$

Commutative	$a+b=b+a$	$2+5=5+2$ $7=7$
Associative	$a+(b+c)=(a+b)+c$	$2+(3+5)=(2+3)+5$ $2+8=5+5$ $10=10$
Identity	$a+0=a\ =\ 0\ +\ a$	$5+0=5=0+5$
Inverse	$a+(-a)=(-a)+a=0$	$6+(-6)=(-6)+6=0$

12. What are the multiplicative properties of real numbers?

Name of the Property	∀ a, b, c ∈ ℝ	Examples
Closure	$ab\in R$	$2\times 5=10\in R$
Commutative	$ab=ba$	$2\times 5=5\times 2$ $10=10$
Associative	$a(bc)=(ab)c$	$2\times(3\times 5)=(2\times 3)\times 5$ $2\times 15=6\times 5$ $30=30$
Identity	$a\times 1=a\ =\ 1\ \times\ a$	$5\times 1=5=1\times 5$
Inverse	$a\times\frac{1}{a}=\frac{1}{a}\times a=1$	$6\times\frac{1}{6}=\frac{1}{6}\times 6=1$

Note:

(i) 0 and 1 are the **additive** and **multiplicative** identities of real numbers, respectively.

(ii) $0\in R$ has no multiplicative inverse.

13. What are the distributive properties of real numbers?

Property	Mathematical Expression
Left Distributive Property of Multiplication over Addition	$a(b+c)=ab+ac$
Left Distributive Property of Multiplication over Subtraction	$a(b-c)=\ ab-ac$
Right Distributive Property of Multiplication over Addition	$(a+b)c=ac+bc$
Right Distributive Property of Multiplication over Subtraction	$(a-b)c=ac-bc$

14. What are the properties of equality of real numbers?

No.	Property Name	Mathematical Expression
i	Reflexive Property	$\forall a\in R,$ $a=a$
ii	Symmetric Property	$\forall a,b\in R,$ $a=b$ $\Longrightarrow b=a$
iii	Transitive Property	$\forall a,b,c\in R,$ $a=b\ \wedge\ b=c$ $\Rightarrow a=c$
iv	Additive Property	$\forall a,b,c\in R,$ $a=b$ $\Rightarrow a+c=b+c$
v	Multiplicative Property	$\forall a,b,c\in R,$ $a=b$ $\Rightarrow ac=bc$

vi	Cancellation Property w.r.t Addition	$\forall a,b,c \in R$, $a+c=b+c$ $\Rightarrow a=b$
vii	Cancellation Property w.r.t Multiplication	$\forall a,b,c \in R$, $ac=bc$ $\Rightarrow a=b$

15. State and explain the Multiplicative Property of order with examples.

No.	Property Name	Mathematical Expression
i	**Trichotomy Property**	$\forall a,b \in R$, Either $a=b$ or $a>b$ or $a<b$
ii	**Transitive Property**	$\forall a,b,c \in R$, ➢ $a>b \wedge b>c \implies a>c$ ➢ $a<b \wedge b<c \implies a<c$
iii	**Additive Property**	$\forall a,b,c \in R$, ➢ $a>b \implies a+c>b+c$ ➢ $a<b \implies a+c<b+c$
iv	**Multiplicative Property**	$\forall a,b,c \in R$, ➢ $a>b \implies ac>bc$ if $c>0$ ➢ $a<b \implies ac<bc$ if $c>0$ ➢ $a>b \implies ac<bc$ if $c<0$ ➢ $a<b \implies ac>bc$ if $c<0$ ➢ $a>b \wedge c>d \implies ac>bd$ ➢ $a<b \wedge c<d \implies ac<bd$
v	**Division Property**	$\forall a,b,c \in R$, ➢ $a>b \implies \frac{a}{c}>\frac{b}{c}$ if $c>0$ ➢ $a<b \implies \frac{a}{c}<\frac{b}{c}$ if $c>0$ ➢ $a>b \implies \frac{a}{c}<\frac{b}{c}$ if $c<0$ ➢ $a<b \implies \frac{a}{c}>\frac{b}{c}$ if $c<0$
vi	**Reciprocal Property**	$\forall a,b,c \in R$ and have same sign ➢ $a>b \implies \frac{1}{a}<\frac{1}{b}$ ➢ $a<b \implies \frac{1}{a}>\frac{1}{b}$

16. Write the Laws of Radicals and Indices.

Laws of Radical	Laws of Indices
(i) $\sqrt[n]{ab}=\sqrt[n]{a}\cdot\sqrt[n]{b}$ **(ii)** $\sqrt[n]{\frac{a}{b}}=\frac{\sqrt[n]{a}}{\sqrt[n]{b}}$ **(iii)** $\sqrt[n]{a^m}=\left(\sqrt[n]{a}\right)^m$ **(iv)** $\left(\sqrt[n]{a}\right)^n=\left(a^{\frac{1}{n}}\right)^n=a$	**(i)** $a^m.a^n=a^{m+n}$ **(ii)** $(a^m)^n=a^{mn}$ **(iii)** $(ab)^n=a^na^n$ **(iv)** $\left(\frac{a}{b}\right)^n=\frac{a^n}{b^n}$ **(v)** $\frac{a^m}{a^n}=a^{m-n}$ **(vi)** $a^0=1$

17. Is 0 a rational number? Explain.

Yes, 0 is a rational number because it can be written in the form $\frac{p}{q}$, where p and q are integers and $q \neq 0$.

For example, $\frac{0}{1}, \frac{0}{2}, \frac{0}{3}$, etc., are valid rational numbers. Since 0 divided by any nonzero integer is always 0, it satisfies the definition of a rational number.

18. What is the historical development of number systems?

1. Sumerians (4500–1900 BCE):

- Used a **base-60 (sexagesimal)** number system.
- Symbols: Cone (1), Bead (10), Large cone (60), etc.
- Used for **counting and astronomy**.

2. Egyptians (3000–2000 BCE):

- Used a **base-10 (decimal)** system.
- Wrote numbers **left to right**, starting with the **largest value**.
- Example: 2525 = 2000 + 500 + 20 + 5.

3. Romans (500 BCE–500 CE):

- Used **Roman numerals**: I (1), V (5), X (10), L (50), C (100), D (500), M (1000)
- Designed for **trade and communication**.

4. Indians (500–1200 CE):

- Invented **zero (0)**.
- Developed the **decimal number system**.
- Introduced **Indo-Arabic numerals** (1–9, 0), which we use today.

5. Arabs (800–1500 CE):

- Spread **Arabic numerals** to Europe.
- **Al-Khwārizmi** introduced **algebra**.
- Preserved and shared mathematical knowledge with the West.

6. Modern Era (1700–present):

- Introduced systems like:
  - **Binary system** (base-2)
  - **Hexadecimal system** (base-16)
- **Arabic numeral system (0–9)** became the global standard.
- Modern number systems expanded to include **real numbers**.

19. What was the contribution of Indian mathematicians to real numbers?

- **Period:** 500–1200 CE
- **Contributions:**
  - Invented zero (0)
  - Refined the decimal (base-10) system
  - Developed Indo-Arabic numerals (basis of today's number system)

20. How did Arabs contribute to modern number systems?

- **Period:** 800–1500 CE
- **Contributions:**
  - Spread **Arabic numerals (0–9)** to Europe
  - **Al-Khwārizmi** introduced **algebra**
  - Helped shape modern mathematics through translations and innovations.