Exercise 1.1

1. Identify each of the following as a rational or irrational number: (i) 2.353535 (ii) $0.\overline{6}$ (iii) $2.236067...$ (iv) $\sqrt{7}$ (v) e (vi) π (vii) $5+\sqrt{11}$(viii) $\sqrt{3}+\sqrt{13}$ (ix) $\frac{15}{4}$ (x) $(2-\sqrt{2})(2+\sqrt{2})$

Sr	Number	Type	Reason
i	2.353535	Rational	Decimal repeats (35 repeats), so it's recurring.
ii	$0.\overline{6}$	Rational	Decimal repeats (6 repeats), so it's recurring.
iii	$2.236067...$	Irrational	Decimal doesn't repeat or end.
iv	$\sqrt{7}$	Irrational	Can't be written as $\frac{p}{q}$, decimal doesn't repeat or end.
v	e	Irrational	Decimal doesn't end or repeat. Known irrational number.
vi	π	Irrational	Decimal doesn't end or repeat. Known irrational number.
vii	$5+\sqrt{11}$	Irrational	Rational + irrational = irrational.
viii	$\sqrt{3}+\sqrt{13}$	Irrational	Irrational + irrational = irrational (in most cases).
ix	$\frac{15}{4}$	Rational	In $\frac{p}{q}$ form where p & q are integers.
x	$(2-\sqrt{2})(2+\sqrt{2})$	Rational	This equals $4-2=2$, which is rational (special product formula).

2. Represent the following numbers on number line: (i) $\sqrt{2}$ (ii) $\sqrt{3}$ (iii) $4\frac{1}{3}$ (iv) $-2\frac{1}{7}$ (v) $\frac{5}{8}$ (vi) $2\frac{3}{4}$

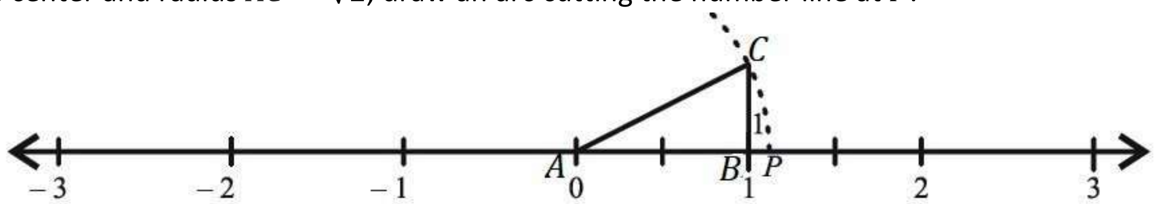
(i) Locating $\sqrt{2} \approx 1.414$

- **a.** Draw a number line with $A=0$ and $B=1$
- **b.** At B, draw a perpendicular $BC = 1\ unit$.
- **c.** Join A to C to form a right-angled ΔABC.

By Pythagoras theorem:

$$(Hypotenuse)^2 = (Base)^2 + (Perpendicular)^2$$
$$(AC)^2 = (AB)^2 + (BC)^2$$
$$(AC)^2 = (1)^2 + (1)^2$$
$$(AC)^2 = 1+1$$
$$(AC)^2 = 2$$
$$\sqrt{(AC)^2} = \sqrt{2}$$
$$AC = \sqrt{2}$$

With A as center and radius $AC = \sqrt{2}$, draw an arc cutting the number line at P.



(ii) Locating $\sqrt{3} \approx 1.732$

Step 1: Construct$\sqrt{2}$

- **a.** Draw a number line with $A=0$ and $B=1$
- **b.** At B, draw a perpendicular $BC = 1\ unit$.
- **c.** Join A to C to form a right-angled ΔABC.

By Pythagoras theorem:

$$(Hypotenuse)^2 = (Base)^2 + (Perpendicular)^2$$
$$(AC)^2 = (AB)^2 + (BC)^2$$
$$(AC)^2 = (1)^2 + (1)^2$$
$$(AC)^2 = 1+1$$

$$(AC)^2 = 2$$
$$\sqrt{(AC)^2} = \sqrt{2}$$
$$AC = \sqrt{2}$$

With A as center and radius $AC = \sqrt{2}$, draw an arc cutting the number line at D.

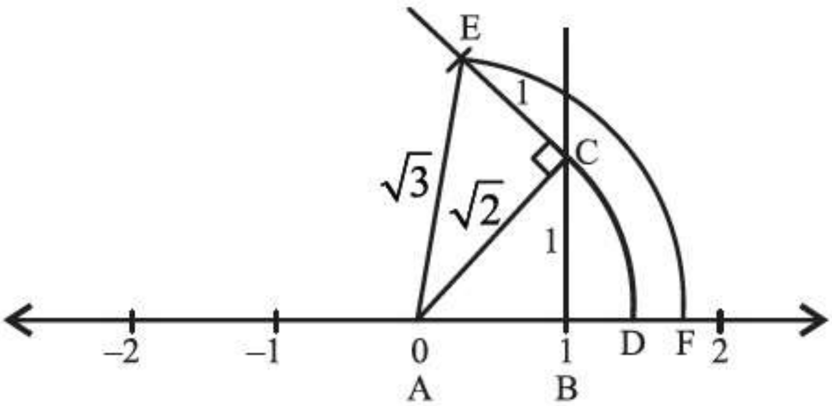
Step 2: Construct $\sqrt{3}$

- **a.** At C, draw a perpendicular $CE = 1\ unit$ (upwards).
- **b.** Join A to E to form a right-angled ΔACE

By Pythagoras theorem:

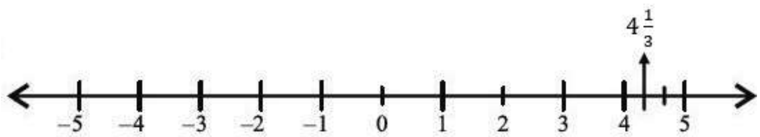
$$(Hypotenuse)^2 = (Base)^2 + (Perpendicular)^2$$
$$(AE)^2 = (AC)^2 + (CE)^2$$
$$(AE)^2 = \left(\sqrt{2}\right)^2 + (1)^2$$
$$(AE)^2 = 2 + 1$$
$$(AE)^2 = 3$$
$$\sqrt{(AE)^2} = \sqrt{3}$$
$$AE = \sqrt{3}$$

With A as center and radius $AE = \sqrt{3}$, draw an arc cutting the number line at F.



(iii) $4\frac{1}{3} = 4.33$

- **a.** Locate 4 and 5 on the number line
- **b.** Divide the space between them into 3 equal parts
- **c.** Mark the point $\frac{1}{3}$ of the way from 4 to 5

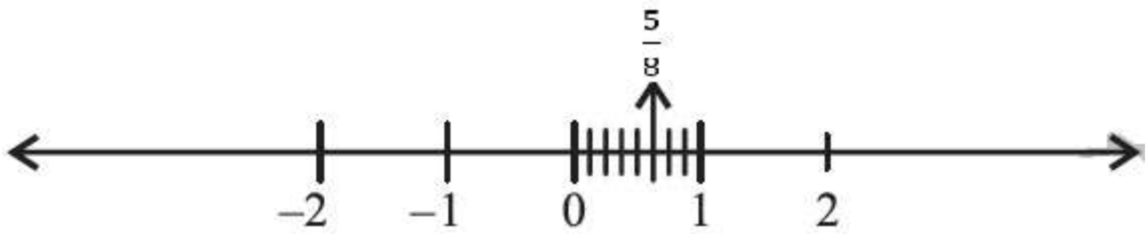


(iv) $-2\frac{1}{7} = -2.143$

- **a.** Locate -2 and -3 on the number line
- **b.** Divide the space between them into 7 equal parts
- **c.** Mark the point $\frac{1}{7}$ of the way from -2 to -3

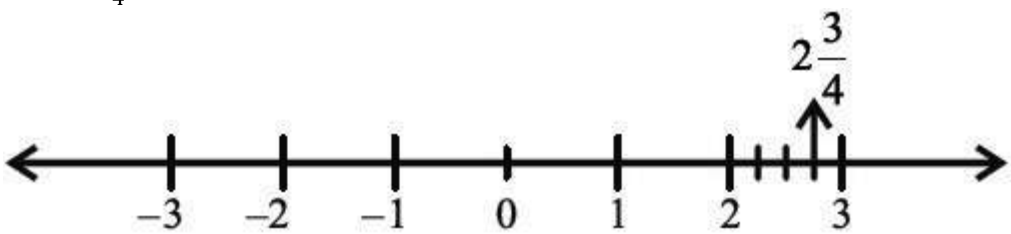
(v) $\frac{5}{8} = 0.625$

a. Locate 0 and 1 on the number line
b. Divide the space into 8 equal parts
c. Mark the 5^{th} division point.



(vi) $2\frac{3}{4} = 2.75$

a. Locate 2 and 3 on the number line
b. Divide the space into 4 equal parts
c. Mark the point $\frac{3}{4}$ of the way from 2 to 3



3. Express the following as a rational number $\frac{p}{q}$ where p and q are integers and $q \neq 0$. (i) $0.\overline{4}$ (ii) $0.\overline{37}$ (iii) $0.\overline{21}$

(i) Let

$$x = 0.\overline{4}$$
$$x = 0.444\ldots \qquad \cdots(i)$$
$$10x = 10(0.444\ldots)$$
$$10x = 4.444\ldots \qquad \cdots(ii)$$

Subtracting (i) from (ii)

$$10x - x = (4.444\ldots) - (0.444\ldots)$$
$$9x = 4$$
$$x = \frac{4}{9}$$

(ii) Let

$$x = 0.\overline{37}$$
$$x = 0.373737\ldots \qquad \cdots(i)$$
$$100x = 100(0.373737\ldots)$$
$$100x = 37.373737\ldots \qquad \cdots(ii)$$

Subtracting (i) from (ii)

$$100x - x = (37.373737\ldots) - (0.373737\ldots)$$
$$99x = 37$$
$$x = \frac{37}{99}$$

(iii) Let

$$x = 0.\overline{21}$$
$$x = 0.212121\ldots \qquad \cdots(i)$$
$$100x = 100(0.212121\ldots)$$
$$100x = 21.212121\ldots \qquad \cdots(ii)$$

Subtracting (i) from (ii)

$$100x - x = (21.212121\ldots) - (0.212121\ldots)$$
$$99x = 21$$
$$x = \frac{21}{99}$$

4. Name the property used in the following: (i) $(\boldsymbol{a}+\boldsymbol{4})+\boldsymbol{b}=\boldsymbol{a}+(\boldsymbol{4}+\boldsymbol{b})$ (ii) $\sqrt{2}+\sqrt{3}=\sqrt{3}+\sqrt{2}$ (iii) $\boldsymbol{x}-\boldsymbol{x}=\boldsymbol{0}$ (iv) $\boldsymbol{a}(\boldsymbol{b}+\boldsymbol{c})=\boldsymbol{ab}+\boldsymbol{ac}$ (v) $16+0=16$ (vi) $100\times1=100$ (vii) $4\times(5\times8)=(4\times5)\times8$ (viii) $\boldsymbol{ab}=\boldsymbol{ba}$

Sr	Expression	Property Name
i	$(a+4)+b=a+(4+b)$	Associative property over addition
ii	$\sqrt{2}+\sqrt{3}=\sqrt{3}+\sqrt{2}$	Commutative property over addition
iii	$x-x=0$	Additive Inverse
iv	$a(b+c)=ab+ac$	Left distributive property of multiplication over addition
v	$16+0=16$	Additive Identity
vi	$100\times1=100$	Multiplicative Identity
vii	$4\times(5\times8)=(4\times5)\times8$	Associative Property under Multiplication
viii	$ab=ba$	Commutative Property under Multiplication

5. Name the property used in the following: (i) $-3<-1\Rightarrow0<2$ (ii) If $\boldsymbol{a}<\boldsymbol{b}$ then $\frac{1}{a}>\frac{1}{b}$ (iii) If $\boldsymbol{a}<\boldsymbol{b}$ then $\boldsymbol{a}+\boldsymbol{c}<\boldsymbol{b}+\boldsymbol{c}$ (iv) If $\boldsymbol{ac}<\boldsymbol{bc}$ and $\boldsymbol{c}>0$ then $\boldsymbol{a}<\boldsymbol{b}$ (v) If $\boldsymbol{ac}<\boldsymbol{bc}$ and $\boldsymbol{c}<0$ then $\boldsymbol{a}>\boldsymbol{b}$ (vi) Either $\boldsymbol{a}>\boldsymbol{b}$ or $\boldsymbol{a}=\boldsymbol{b}$ or $\boldsymbol{a}<\boldsymbol{b}$

Sr	Expression	Property Name
i	$-3<-1\Rightarrow0<2$	Additive Property of Inequality
ii	If $a<b$ then $\frac{1}{a}>\frac{1}{b}$	Reciprocal Property
iii	If $a<b$ then $a+c<b+c$	Additive Property (of order)
iv	If $ac<bc$ and $c>0$ then $a<b$	Multiplicative Property (when $c>0$)
v	If $ac<bc$ and $c<0$ then $a>b$	Multiplicative Property (when $c<0$)
vi	Either $a>b$ or $a=b$ or $a<b$	Trichotomy Property

6. Find two rational numbers between: (i) $\frac{1}{3}$ and $\frac{1}{4}$ (ii) 3 and 4 (iii) $\frac{3}{5}$ and $\frac{4}{5}$

(i) $\frac{1}{3}$ and $\frac{1}{4}$

$$1^{st}\ rational\ number = \left(\frac{1}{3}+\frac{1}{4}\right)\div2$$
$$= \left(\frac{4+3}{12}\right)\times\frac{1}{2}$$
$$= \frac{7}{12}\times\frac{1}{2}$$
$$= \frac{7}{24}$$

$$2^{nd}\ rational\ number = \left(\frac{1}{3}+\frac{7}{24}\right)\div2$$
$$= \left(\frac{8+7}{24}\right)\times\frac{1}{2}$$
$$= \frac{15}{24}\times\frac{1}{2}$$
$$= \frac{15}{48}$$

3	3,24
2	1,8
2	1,4
2	1,2
	1,1

(ii) 3 and 4

$$1^{st}\ rational\ number = (3+4) \div 2$$
$$= (7) \times \frac{1}{2}$$
$$= \frac{7}{2}$$

$$2^{nd}\ rational\ number = \left(3 + \frac{7}{2}\right) \div 2$$
$$= \left(\frac{6+7}{2}\right) \times \frac{1}{2}$$
$$= \frac{13}{2} \times \frac{1}{2}$$
$$= \frac{13}{4}$$

(iii) $\frac{3}{5}$ and $\frac{4}{5}$

$$1^{st}\ rational\ number = \left(\frac{3}{5} + \frac{4}{5}\right) \div 2$$
$$= \left(\frac{3+4}{5}\right) \times \frac{1}{2}$$
$$= \frac{7}{5} \times \frac{1}{2}$$
$$= \frac{7}{10}$$

2	5 , 10
5	5 , 5
	1 , 1

$$2^{nd}\ rational\ number = \left(\frac{3}{5} + \frac{7}{10}\right) \div 2$$
$$= \left(\frac{6+7}{10}\right) \times \frac{1}{2}$$
$$= \frac{13}{10} \times \frac{1}{2}$$
$$= \frac{13}{20}$$