

Review Exercise 1

2. if $a = \frac{3}{2}$, $b = \frac{5}{3}$ and $c = \frac{7}{5}$ then verify that (i) $a(b + c) = ab + ac$ (ii) $(a + b)c = ac + bc$

(i) $a(b + c) = ab + ac$

$$\begin{aligned} a(b + c) &= ab + ac \\ \frac{3}{2} \left(\frac{5}{3} + \frac{7}{5} \right) &= \left(\frac{3}{2} \right) \left(\frac{5}{3} \right) + \left(\frac{3}{2} \right) \left(\frac{7}{5} \right) \\ \frac{3}{2} \left(\frac{25 + 21}{15} \right) &= \frac{15}{6} + \frac{21}{10} \\ \frac{3}{2} \left(\frac{46}{15} \right) &= \frac{75 + 63}{30} \\ \frac{138}{30} &= \frac{138}{30} \quad (\text{Proved}) \end{aligned}$$

| | |
|---|------|
| 2 | 6,10 |
| 3 | 3,5 |
| 5 | 1,5 |
| | 1,1 |

(ii) $(a + b)c = ac + bc$

$$\begin{aligned} (a + b)c &= ac + bc \\ \left(\frac{3}{2} + \frac{5}{3} \right) \frac{7}{5} &= \left(\frac{3}{2} \right) \left(\frac{7}{5} \right) + \left(\frac{5}{3} \right) \left(\frac{7}{5} \right) \\ \left(\frac{9 + 10}{6} \right) \frac{7}{5} &= \frac{21}{10} + \frac{35}{15} \\ \left(\frac{19}{6} \right) \frac{7}{5} &= \frac{63 + 70}{30} \\ \frac{133}{30} &= \frac{133}{30} \quad (\text{Proved}) \end{aligned}$$

| | |
|---|-------|
| 2 | 10,15 |
| 3 | 5,15 |
| 5 | 5,5 |
| | 1,1 |

3. if $a = \frac{4}{3}$, $b = \frac{5}{2}$ and $c = \frac{7}{4}$ then verify the associative property of real numbers w.r.t addition and multiplication.

Associative property w.r.t addition

$$\begin{aligned} a + (b + c) &= (a + b) + c \\ \frac{4}{3} + \left(\frac{5}{2} + \frac{7}{4} \right) &= \left(\frac{4}{3} + \frac{5}{2} \right) + \frac{7}{4} \\ \frac{4}{3} + \left(\frac{10 + 7}{4} \right) &= \left(\frac{8 + 15}{6} \right) + \frac{7}{4} \\ \frac{4}{3} + \frac{17}{4} &= \frac{23}{6} + \frac{7}{4} \\ \frac{16 + 51}{12} &= \frac{46 + 21}{12} \\ \frac{67}{12} &= \frac{67}{12} \quad (\text{Proved}) \end{aligned}$$

| | |
|---|-----|
| 2 | 2,4 |
| 2 | 1,2 |
| | 1,1 |

| | |
|---|-----|
| 2 | 4,6 |
| 2 | 2,3 |
| 3 | 1,3 |
| | 1,1 |

Associative property w.r.t multiplication

$$\begin{aligned} a(bc) &= (ab)c \\ \frac{4}{3} \left(\frac{5}{2} \times \frac{7}{4} \right) &= \left(\frac{4}{3} \times \frac{5}{2} \right) \frac{7}{4} \\ \frac{4}{3} \left(\frac{35}{8} \right) &= \left(\frac{20}{6} \right) \frac{7}{4} \\ \frac{140}{24} &= \frac{140}{24} \quad (\text{Proved}) \end{aligned}$$

4. Is 0 a rational number? Explain.

Yes, zero (0) is a rational number. It satisfies the definition of rational numbers. For example, $\frac{0}{2}$, $\frac{0}{-9}$ both are rational numbers.

5. State trichotomy property of real numbers.

$\forall a, b \in \mathbb{R}$, either $a = b$ or $a > b$ or $a < b$

6. Find two rational numbers between 4 and 5.

$$\begin{aligned} 1^{\text{st}} \text{ rational number} &= (4 + 5) \div 2 \\ &= (9) \times \frac{1}{2} \\ &= \frac{9}{2} \end{aligned}$$

$$\begin{aligned} 2^{\text{nd}} \text{ rational number} &= \left(4 + \frac{9}{2} \right) \div 2 \\ &= \left(\frac{8 + 9}{2} \right) \times \frac{1}{2} \\ &= \frac{17}{2} \times \frac{1}{2} \\ &= \frac{17}{4} \end{aligned}$$

7. Simplify the following:

(i) $\sqrt[5]{\frac{x^{15}y^{35}}{z^{20}}}$

$$\begin{aligned} &\sqrt[5]{\frac{x^{15}y^{35}}{z^{20}}} \\ &= \left(\frac{x^{15}y^{35}}{z^{20}} \right)^{\frac{1}{5}} \\ &= \frac{(x^{15})^{\frac{1}{5}}(y^{35})^{\frac{1}{5}}}{(z^{20})^{\frac{1}{5}}} \\ &= \frac{x^3y^7}{z^4} \end{aligned}$$

(ii) $\sqrt[3]{(27)^{2x}}$

$$\begin{aligned} &\sqrt[3]{(27)^{2x}} \\ &= [(27)^{2x}]^{1/3} \\ &= (3^3)^{2x/3} \\ &= 3^{2x} \end{aligned}$$

(iii) $\frac{6(3)^{n+2}}{3^{n+1}-3^n}$

$$\begin{aligned} &\frac{6(3)^{n+2}}{3^{n+1}-3^n} \\ &= \frac{6 \cdot 3^n \cdot 3^2}{3^n \cdot 3^1 - 3^n} \\ &= \frac{6 \cdot 3^n \cdot 9}{3^n(3 - 1)} \end{aligned}$$

$$= \frac{54}{2}$$

$$= 27$$

8. The sum of three consecutive odd integers is 51. Find the three integers.

$$\text{Let first integer} = x$$

$$\text{Second integer} = x + 2$$

$$\text{Third integer} = x + 4$$

According to question

$$x + x + 2 + x + 4 = 51$$

$$3x + 6 = 51$$

$$3x = 51 - 6$$

$$3x = 45$$

$$x = \frac{45}{3}$$

$$x = 15$$

Hence

$$\text{First integer} = x = 15$$

$$\text{Second integer} = x + 2 = 15 + 2 = 17$$

$$\text{Third integer} = x + 4 = 15 + 4 = 19$$

9. Abdullah picked up 96 balls and placed them into two buckets. One bucket has twenty-eight more balls than the other bucket. How many balls were in each bucket?

$$\text{Let balls in first bucket} = x$$

$$\text{Balls in second bucket} = x + 28$$

$$\text{Total balls} = 96$$

According to question

$$x + x + 28 = 96$$

$$2x + 28 = 96$$

$$2x = 96 - 28$$

$$2x = 68$$

$$x = \frac{68}{2}$$

$$x = 34$$

Hence

$$\text{Balls in first bucket} = x = 34$$

$$\text{Balls in second bucket} = x + 28$$

$$= 34 + 28$$

$$= 62$$

10. Salma invested Rs. 3,50,000 in a bank, which paid simple profit at the rate of $7\frac{1}{4}\%$ per annum. After 2 years, the rate was increased to 8% per annum. Find the amount she had at the end of 7 years.

For first 2 years:

$$\text{Principal Amount} = 350000 \text{ Rs}$$

$$\text{Rate} = 7\frac{1}{4}\%$$

$$= 7.25\%$$

$$\text{Time} = 2 \text{ years}$$

$$\text{Profit} = \frac{\text{Principal Amount} \times \text{time} \times \text{rate}}{100}$$

$$P_1 = \frac{350000 \times 2 \times 7.25}{100}$$

$$P_1 = 50750 \text{ Rs.}$$

For Next 5 years:

$$\text{Principal Amount} = 350000 \text{ Rs}$$

$$\text{Rate} = 8\%$$

$$\text{Time} = 2 \text{ years}$$

$$\text{Profit} = \frac{\text{Principal Amount} \times \text{time} \times \text{rate}}{100}$$

$$P_2 = \frac{350000 \times 2 \times 8}{100}$$

$$P_2 = 140000 \text{ Rs.}$$

At end of 7 years:

$$\begin{aligned} \text{Total Amount} &= \text{Principal Amount} + P_1 + P_2 \\ &= 350000 + 50750 + 140000 \\ &= 540750 \text{ Rs.} \end{aligned}$$

Muhammad Tayyab (GHS Christian Daska)