Exercise 2.4

1. Without using a calculator, evaluate the following:

(i) $\log_2 18 - \log_2 9$

$$\log_2 18 - \log_2 9$$
$$= \log_2 \frac{18}{9}$$
$$= \log_2 2$$
$$= 1 \qquad \because \log_a a = 1$$

(ii) $\log_2 64 - \log_2 2$

$$\log_2 64 - \log_2 2$$
$$= \log_2 64 \times 2$$
$$= \log_2 128$$
$$= \log_2 2^7$$
$$= 7\log_2 2$$
$$= 7(1) \qquad \because \log_a a = 1$$
$$= 7$$

(iii) $\frac{1}{3}\log_3 8 - \log_3 18$

$$\frac{1}{3}\log_3 8 - \log_3 18$$
$$= \frac{1}{3}\log_3 2^3 - \log_3 2 \cdot 3^2$$
$$= \frac{\cancel{3}}{\cancel{3}}\log_3 2 - [\log_3 2 + \log_3 3^2]$$
$$= \cancel{\log_3 2} - \cancel{\log_3 2} - 2\log_3 3$$
$$= -2\log_3 3$$
$$= -2(1) \qquad \because \log_a a = 1$$
$$= -2$$

(iv) $2\log 2 + \log 25$

$$2\log 2 + \log 25$$
$$= 2\log 2 + \log 5^2$$
$$= 2\log 2 + 2\log 5$$
$$= 2[\log 2 + \log 5]$$
$$= 2\log 2 \times 5$$
$$= 2\log 10$$
$$= 2\log_{10} 10$$
$$= 2(1)$$
$$= 2$$

(v) $\frac{1}{3}\log_3 64 + 2\log_5 25$

$$\frac{1}{3}\log_4 64 + 2\log_5 25$$
$$= \frac{1}{3}\log_4 4^3 + 2\log_5 5^2$$
$$= \frac{3}{3}\log_4 4 + 2 \times 2\log_5 5$$
$$= \log_4 4 + 4\log_5 5$$
$$= 1 + 4(1)$$
$$= 1 + 4$$
$$= 5$$

(vi) $\log_3 12 + \log_3 0.25$

$$\log_3 12 + \log_3 0.25$$
$$= \log_3(12 \times 0.25)$$
$$= \log_3\left(12 \times \frac{25}{100}\right)$$
$$= \log_3 3$$
$$= 1$$

2. Write the following as a single logarithm:

(i) $\frac{1}{2}\log 25 - 2\log 3$

$$\frac{1}{2}\log 25 - 2\log 3$$
$$= \log(5^2)^{\frac{1}{2}} - \log 3^2$$
$$= \log 5 - \log 9$$
$$= \log\frac{5}{9}$$

(ii) $\log 9 - \log\frac{1}{3}$

$$\log 9 - \log\frac{1}{3}$$
$$= \log\frac{9}{1/3}$$
$$= \log\frac{9 \times 3}{1}$$
$$= \log 27$$

(iii) $\log_5 b^2 \cdot \log_a 5^3$

$$\log_5 b^2 \cdot \log_a 5^3$$
$$= 2\log_5 b \cdot 3\log_a 5$$
$$= 6\log_5 b \cdot \log_a 5$$
$$= 6\frac{\log b}{\log 5} \cdot \frac{\log 5}{\log a} \qquad \because \log_b x = \frac{\log_a x}{\log_a b}$$
$$= 6\frac{\log b}{\log a}$$
$$= 6\log_a b \qquad \because \frac{\log_a x}{\log_a b} = \log_b x$$

(iv) $2\log_3 x + \log_3 y$

$$2\log_3 x + \log_3 y$$
$$= \log_3 x^2 + \log_3 y$$
$$= \log_3 x^2 y$$

(v) $4\log_5 x - \log_5 y + \log_5 z$

$$4\log_5 x - \log_5 y + \log_5 z$$
$$= \log_5 x^4 - \log_5 y + \log_5 z$$
$$= \log_5 \frac{x^4 z}{y}$$

(vi) $2\ln a + 3\ln b - 4\ln c$

$$2\ln a + 3\ln b - 4\ln c$$

$$= \ln a^2 + \ln b^3 - \ln c^4$$
$$= \ln \frac{a^2 b^3}{c^4}$$

3. Expand the following using laws of logarithms:

(i) $\log\left(\frac{11}{5}\right)$

$$\log\left(\frac{11}{5}\right)$$
$$= \log 11 - \log 5$$

(ii) $\log_5 \sqrt{8a^6}$

$$\ln \frac{a^2 b}{c}$$
$$= \ln a^2 + \ln b - \ln c$$
$$= 2\ln a + \ln b - \ln c$$

(iii) $\ln \frac{a^2 b}{c}$

$$\ln \frac{a^2 b}{c}$$
$$= \ln a^2 + \ln b - \ln c$$
$$= 2\ln a + \ln b - \ln c$$

(iv) $\log\left(\frac{xy}{z}\right)^{\frac{1}{9}}$

$$\log\left(\frac{xy}{z}\right)^{\frac{1}{9}}$$
$$= \frac{1}{9}\left[\log\left(\frac{xy}{z}\right)\right]$$
$$= \frac{1}{9}[\log x + \log y - \log z]$$

(v) $\ln \sqrt[3]{16x^3}$

$$\ln \sqrt[3]{16x^3}$$
$$= \ln(16x^3)^{\frac{1}{3}}$$
$$= \frac{1}{3}[\ln 2^4 x^3]$$
$$= \frac{1}{3}[\ln 2^4 + \ln x^3]$$
$$= \frac{1}{3}[4\ln 2 + 3\ln x]$$
$$= \frac{4}{3}\ln 2 + \frac{3}{3}\ln x$$
$$= \frac{4}{3}\ln 2 + \ln x$$

(vi) $\log_2\left(\frac{1-a}{b}\right)^5$

$$\log_2\left(\frac{1-a}{b}\right)^5$$
$$= 5\log_2\left(\frac{1-a}{b}\right)$$
$$= 5[\log_2(1-a) - \log_2 b]$$

4. Find the value of x in the following equations:

(i) $\log 2 + \log x = 1$

$$\log 2 + \log x = 1$$
$$\log 2x = 1$$
$$\log_{10} 2x = 1$$
$$10^1 = 2x$$
$$\frac{10}{2} = x$$
$$5 = x$$
$$\boldsymbol{x = 5}$$

(ii) $\log_2 x + \log_2 8 = 5$

$$\log_2 x + \log_2 8 = 5$$
$$\log_2 8x = 5$$
$$2^5 = 8x$$
$$32 = 8x$$
$$\frac{32}{8} = x$$
$$4 = x$$
$$\boldsymbol{x = 4}$$

(iii) $(81)^x = (243)^{x+2}$

$$(81)^x = (243)^{x+2}$$
$$(3^4)^x = (3^5)^{x+2}$$
$$3^{4x} = 3^{5x+10}$$
$$\Rightarrow 4x = 5x + 10$$
$$4x - 5x = 10$$
$$-x = 10$$
$$\boldsymbol{x = -10}$$

(iv) $\left(\frac{1}{27}\right)^{x-6} = 27$

$$\left(\frac{1}{27}\right)^{x-6} = 27$$
$$(27^{-1})^{x-6} = 27$$
$$27^{-x+6} = 27^1$$
$$\Rightarrow -x + 6 = 1$$
$$-x = 1 - 6$$
$$-x = -5$$
$$\boldsymbol{x = 5}$$

(v) $\log(5x - 10) = 2$

$$\log(5x - 10) = 2$$
$$\log_{10}(5x - 10) = 2$$
$$10^2 = 5x - 10$$
$$100 = 5x - 10$$
$$100 + 10 = 5x$$
$$110 = 5x$$
$$\frac{110}{5} = x$$
$$22 = x$$
$$\boldsymbol{x = 22}$$

(vi) $\log_2(x+1)-\log_2(x-4)=2$

$$\log_2(x+1)-\log_2(x-4)=2$$
$$\log_2\left(\frac{x+1}{x-4}\right)=2$$
$$2^2=\frac{x+1}{x-4}$$
$$4=\frac{x+1}{x-4}$$
$$4(x-4)=x+1$$
$$4x-16=x+1$$
$$4x-x=1+16$$
$$3x=17$$
$$x=\frac{17}{3}$$
$$\boldsymbol{x=5\frac{2}{3}}$$

5. Find the values of the following with the help of logarithm table:

(i) $\frac{3.68\times4.21}{5.234}$

Let

$$x=\frac{3.68\times4.21}{5.234}$$
$$\log x=\log\left(\frac{3.68\times4.21}{5.234}\right)$$
$$\log x=\log 3.68+\log 4.21-\log 5.234$$
$$\log x=0.5658\ +\ 0.6243\ -\ 0.7188$$
$$\log x=0.4713$$
$$Anti\log(\log x)=Anti\log(0.4713)$$
$$\boldsymbol{x=2.960}$$

(ii) $4.67\times2.11\times2.397$

Let

$$x=4.67\ \times\ 2.11\ \times\ 2.397$$
$$\log x=\log(4.67\ \times\ 2.11\ \times\ 2.397)$$
$$\log x=\log 4.67+\log 2.11+\log\ 2.397$$
$$\log x=0.6693\ +\ 0.3243\ +\ 0.3797$$
$$\log x=1.3733$$
$$Anti\log(\log x)=Anti\log(1.3733)$$
$$\boldsymbol{x=23.62}$$

(iii) $\frac{(20.46)^2\times(2.4122)}{754.3}$

Let

$$x=\frac{(20.46)^2\times(2.4122)}{754.3}$$
$$\log x=\log\left[\frac{(20.46)^2\times(2.4122)}{754.3}\right]$$
$$\log x=\log(20.46)^2+\log 2.4122-\log 754.3$$
$$\log x=2\log 20.46+\log 2.4122-\log 754.3$$
$$\log x=2(1.3109)+\ 0.3824\ -\ 2.8775$$
$$\log x=2.6218\ +\ 0.3824\ -\ 2.8775$$
$$\log x=0.1267$$
$$Anti\log(\log x)=Anti\log(0.1267)$$
$$\boldsymbol{x=1.338}$$

(iv) $\frac{\sqrt[3]{9.364}\times21.64}{3.21}$

Let

$$x=\frac{\sqrt[3]{9.364}\times21.64}{3.21}$$

$$\log x = \log\left[\frac{\sqrt[3]{9.364} \times 21.64}{3.21}\right]$$
$$\log x = \log \sqrt[3]{9.364} + \log 21.64 - \log 3.21$$
$$\log x = \log(9.364)^{1/3} + \log 21.64 - \log 3.21$$
$$\log x = \frac{1}{3}\log 9.364 + \log 21.64 - \log 3.21$$
$$\log x = \frac{1}{3}(0.9715) + 1.335 - 0.5065$$
$$\log x = 0.3238 + 1.335 - 0.5065$$
$$\log x = 1.1523$$
$$Anti\log(\log x) = Anti\log(1.1523)$$
$$\boldsymbol{x = 14.20}$$

6. The formula to measure the magnitude of earthquakes is given by $M = \log_{10}\left(\frac{A}{A_\circ}\right)$. If amplitude (A) is $10,000$ and reference amplitude $(A_\circ)$ is 10, what is the magnitude of the earthquake?

Since

$$M = \log_{10}\left(\frac{A}{A_\circ}\right)$$
$$\text{Put } A = 10000 \text{ and } A_\circ = 10$$
$$M = \log_{10}\left(\frac{10000}{10}\right)$$
$$M = \log_{10}(1000)$$
$$M = \log_{10} 10^3$$
$$M = 3\log_{10} 10$$
$$M = 3(1)$$
$$\boldsymbol{M = 3}$$

7. Abdullah invested Rs. 100,000 in a saving scheme and gains interest at the rate of 5% per annum so that the total value of this investment after t years is Rs y. This is modelled by an equation $y = 1,00,000(1.05)^t, t \geq 0$. Find after how many years the investment will be doubled?

$$Initial\ investment\ amount = 100000\ Rs.$$
$$Double\ the\ investment = y = 200000\ Rs$$
$$Time = t = ?$$

Since

$$y = 100000(1.05)^t$$
$$200000 = 100000(1.05)^t$$
$$\frac{200000}{100000} = (1.05)^t$$
$$2 = (1.05)^t$$
$$\log 2 = \log(1.05)^t$$
$$\log 2 = t\log 1.05$$
$$\frac{\log 2}{\log 1.05} = t$$
$$\frac{0.3010}{0.0212} = t$$
$$14.2 = t$$
$$\boldsymbol{t = 14.2\ years}$$

8. Huria is hiking up a mountain where the temperature (T) decreases by 3% (or a factor of 0.97) for every 100 meters gained in altitude. The initial temperature (T_i) at sea level is 20℃. Using the formula $T = T_i \times 0.97^{h/100}$, calculate the temperature at an altitude (h) of 500 meters.

$$Initial\ Temperature = T_i = 20℃$$
$$Altitude = h = 500\ m$$
$$Final\ Temperature = T = \ ?$$

Since

$$T = T_i \times 0.97^{h/100}$$
$$\text{Put } T_i = 20 \text{ and } h = 500$$
$$T = 20 \times 0.97^{500/100}$$
$$T = 20 \times 0.97^5$$
$$\log T = \log[20 \times 0.97^5]$$
$$\log T = \log 20 + \log 0.97^5$$
$$\log T = \log 20 + 5 \log 0.97$$
$$\log T = 1.3010 + 5(-0.0132)$$
$$\log T = 1.3010 - 0.066$$
$$\log T = 1.235$$
$$Anti \log(\log T) = Anti \log 1.235$$
$$\boldsymbol{T = 17.18℃}$$