Exercise 3.3

1. For $A = \{1, 2, 3, 4\}$ find the following relations in A. State the domain and range of each relation.

$$A \times A = \{1,2,3,4\} \times \{1,2,3,4\}$$

$$A \times A = \{(1,1),(1,2),(1,3),(1,4),(2,1),(2,2),(2,3),(2,4),(3,1),(3,2),(3,3),(3,4),(4,1),(4,2),(4,3),(4,4)\}$$

(i) $\{(x, y) \mid y = x\}$

Let

$$R_1 = \{(x, y) \mid y = x\}$$
$$R_1 = \{(1,1),(2,2),(3,3),(4,4)\}$$
$$Dom\ (R_1) = \{1,2,3,4\}$$
$$Range\ (R_1) = \{1,2,3,4\}$$

(ii) $\{(x, y) \mid y + x = 5\}$

Let

$$R_2 = \{(x, y) \mid y + x = 5\}$$
$$R_2 = \{(1,4),(2,3),(3,2),(4,1)\}$$
$$Dom\ (R_2) = \{1,2,3,4\}$$
$$Range\ (R_2) = \{1,2,3,4\}$$

(iii) $\{(x, y) \mid x + y < 5\}$

Let

$$R_3 = \{(x, y) \mid x + y < 5\}$$
$$R_3 = \{(1,1),(1,2),(1,3),(2,1),(2,2),(3,1)\}$$
$$Dom\ (R_3) = \{1,2,3\}$$
$$Range\ (R_3) = \{1,2,3\}$$

(iii) $\{(x, y) \mid x + y > 5\}$

Let

$$R_3 = \{(x, y) \mid x + y > 5\}$$
$$R_3 = \{(2,4),(3,3),(3,4),(4,2),(4,3),(4,4)\}$$
$$Dom\ (R_3) = \{2,3,4\}$$
$$Range\ (R_3) = \{2,3,4\}$$

2. Which of the following diagrams represent functions and of which type?

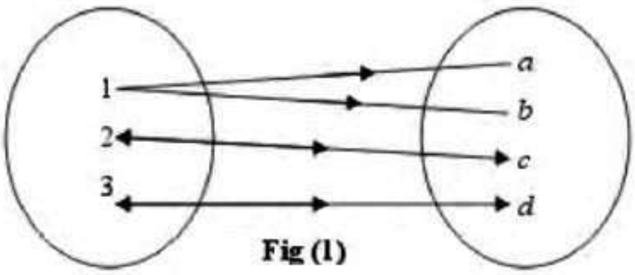

Fig (1)

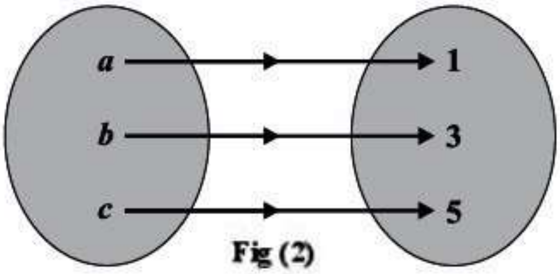

Fig (2)

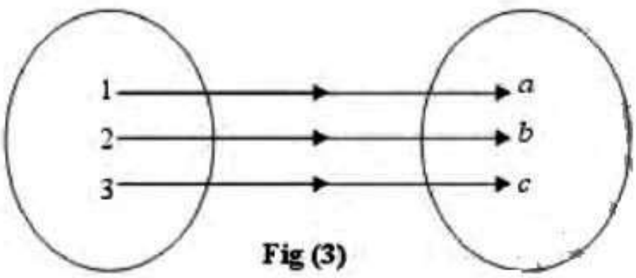

Fig (3)

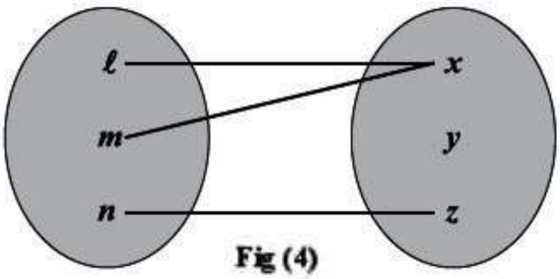

Fig (4)

(i) $\{(1,a)(1,b),(2,c)(c,2),(3,d),(d,3)\}$ Since domain should not be repeated, but here domain is repeated so it is not a function.

(ii) It is a bijective function (one-one and onto)

(iii) It is a bijective function (one-one and onto)

(iv) It is into function.

3. If $g(x) = 3x + 2$ and $h(x) = x^2 + 1$, then find:

(i) $g(0)$

Since

$$g(x) = 3x + 2$$

Put $x = 0$

$$g(0) = 3(0) + 2$$
$$= 0 + 2$$
$$= 2$$

(ii) $g(-3)$

Since

$$g(x) = 3x + 2$$

Put $x = -3$

$$g(-3) = 3(-3) + 2$$
$$= -9 + 2$$
$$= -7$$

(iii) $g\left(\frac{2}{3}\right)$

Since

$$g(x) = 3x + 2$$

Put $x = \frac{2}{3}$

$$g\left(\frac{2}{3}\right) = 3\left(\frac{2}{3}\right) + 2$$
$$= 2 + 2$$
$$= 4$$

(iv) $h(1)$

Since

$$h(x) = x^2 + 1$$

Put $x = 1$

$$h(1) = (1)^2 + 1$$
$$= 1 + 1$$
$$= 2$$

(v) $h(-4)$

Since

$$h(x) = x^2 + 1$$

Put $x = -4$

$$h(-4) = (-4)^2 + 1$$
$$= 16 + 1$$
$$= 17$$

(v) $h\left(-\frac{1}{2}\right)$

Since

$$h(x) = x^2 + 1$$

Put $x = -\frac{1}{2}$

$$h\left(-\frac{1}{2}\right) = \left(-\frac{1}{2}\right)^2 + 1$$
$$= \frac{1}{4} + 1$$
$$= \frac{1+4}{4}$$
$$= \frac{5}{4}$$

4. Given that $f(x) = ax + b + 1$, where a and b are constant numbers. If $f(3) = 8$ and $f(6) = 14$, then find the values of a and b.

Since

$$f(x) = ax + b + 1$$

Put $x = 3$

$$f(3) = a(3) + b + 1$$
$$8 = 3a + b + 1 \qquad \because f(3) = 8$$
$$8 - 1 = 3a + b$$
$$7 = 3a + b \quad \dots \ (i)$$

Now

$$f(x) = ax + b + 1$$

Put $x = 6$

$$f(6) = a(6) + b + 1$$
$$14 = 6a + b + 1 \qquad \because f(6) = 14$$
$$14 - 1 = 6a + b$$
$$13 = 6a + b \quad \dots \ (ii)$$

Subtract equation (i) from equation (ii)

$$13 = 6a + b$$
$$\pm 7 = \pm 3a \pm b$$

$$6 = 3a$$
$$\frac{6}{3} = a$$
$$\mathbf{2 = a}$$

Putting $a = 2$ in equation (i)

$$7 = 3(2) + b$$
$$7 = 6 + b$$
$$7 - 6 = b$$
$$\mathbf{1 = b}$$

5. Given that $g(x) = ax + b + 5$, where a and b are constant numbers. If $g(-1) = 0$ and $g(2) = 10$, find the values of a and b.

Since

$$g(x) = ax + b + 5$$

Put $x = -1$

$$g(-1) = a(-1) + b + 5$$
$$0 = -a + b + 5 \quad \because g(-1) = 0$$
$$-5 = -a + b \quad \dots \ (i)$$

Now

$$g(x) = ax + b + 5$$

Put $x = 2$

$$g(2) = a(2) + b + 5$$
$$10 = 2a + b + 5 \quad \because g(2) = 10$$
$$10 - 5 = 2a + b$$
$$5 = 2a + b \quad \dots \ (ii)$$

Subtract equation (i) from equation (ii)

$$5 = 2a + b$$
$$\mp 5 = \mp a \pm b$$
$$10 = 3a$$
$$\mathbf{\frac{10}{3} = a}$$

Putting $a = \frac{10}{3}$ in equation (i)

$$-5 = -\frac{10}{3} + b$$
$$\frac{10}{3} - 5 = b$$
$$\frac{10 - 15}{3} = b$$
$$\mathbf{\frac{-5}{3} = b}$$

6. Consider the function defined by $f(x) = 5x + 2$. If $f(x) = 32$, find the x value.

Since

$$f(x) = 5x + 2$$

Put $f(x) = 32$

$$32 = 5x + 2$$
$$32 - 2 = 5x$$
$$30 = 5x$$
$$\frac{30}{5} = x$$
$$6 = x$$
$$\mathbf{x = 6}$$

7. Consider the function $f(x) = cx^2 + d$, where c and d are constant numbers. If $f(1) = 6$ and $f(-2) = 10$, then find the values of c and d.

Since

$$f(x) = cx^2 + d$$

Put $x = 1$

$$f(1) = c(1)^2 + d$$
$$6 = c(1) + d \quad \because f(1) = 6$$
$$6 = c + d \quad \dots \ (i)$$

Now

$$f(x) = cx^2 + d$$

Put $x = -2$

$$f(-2) = c(-2)^2 + d$$
$$10 = c(4) + d \quad \because f(-2) = 10$$
$$10 = 4c + d \quad \dots \ (ii)$$

Subtract equation (i) from equation (ii)

$$10 = 4c + d$$
$$\pm 6 = \pm c \pm d$$
$$4 = 3c$$
$$\mathbf{\frac{4}{3} = c}$$

Putting $c = \frac{4}{3}$ in equation (i)

$$6 = \frac{4}{3} + d$$
$$6 - \frac{4}{3} = d$$
$$\frac{18 - 4}{3} = d$$
$$\mathbf{\frac{14}{3} = d}$$