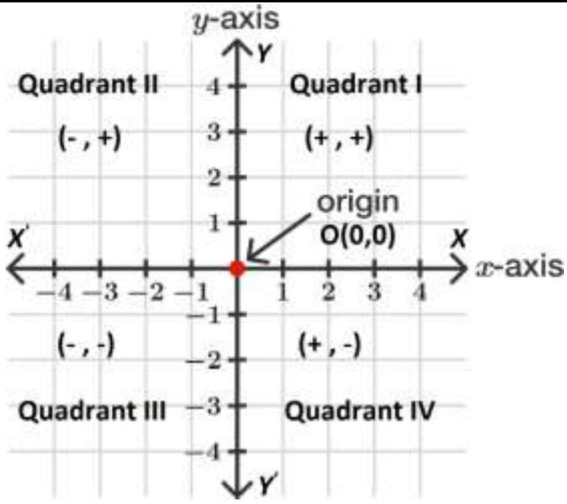
1. Define coordinate plane (Cartesian plane).

The plane formed by two straight lines perpendicular to each other is called coordinate plane and the lines XOX' and YOY' are called coordinate axes.



2. Define order pair.

An ordered pair of real numbers x and y is a pair (x, y) in which elements are written in specific order.

Note: $(\boldsymbol{x}, \boldsymbol{y}) \neq (\boldsymbol{y}, \boldsymbol{x})$

3. Define origin.

The point of intersection of two coordinate axes is called origin.

4. Differentiate between abscissa and ordinate.

The $x\text{-}coordinate$ of the point is called abscissa of the point $P(X, Y)$ and the $y\text{-}coordinate$ is called its ordinate.

5. What are the coordinates of points lying on the axes?

- On the $\boldsymbol{x\text{-}axis}$: $(a, 0)$
- On the $\boldsymbol{y\text{-}axis}$: $(0, b)$

6. Define distance formula.

If $A(x_1, y_1)$ and $B(x_2, y_2)$ are two points and d is the distance between them, then

$$d = |\overline{AB}| = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2} \qquad d \geq 0$$

Note: $|\overline{AB}|$ stands for $m\overline{AB}$

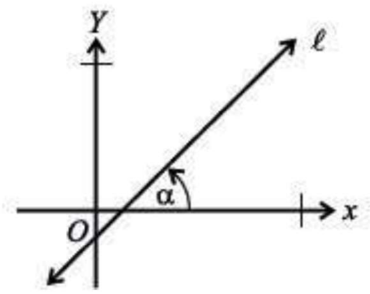
7. Define midpoint formula.

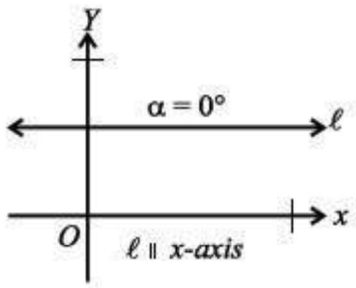
If $A(x_1, y_1)$ and $B(x_2, y_2)$ are two points in the plane, then the mid-point $M(x, y)$ of line segment $\overline{AB}$ is

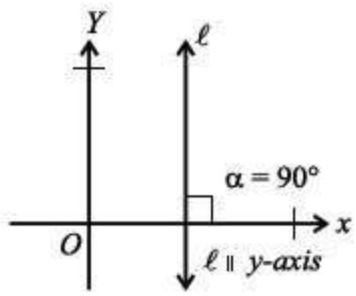
$$M(x, y) = \left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right)$$

8. What is the inclination of a line?

The inclination of a line is the angle α $(0^\circ < \alpha < 180^\circ)$ measured counterclockwise from the positive $x\text{-}axis$ to a non-horizontal straight line l.







- If the line $\boldsymbol{\ell}$ **is parallel to the** $\boldsymbol{x\text{-}axis}$, then $\boldsymbol{\alpha} = \boldsymbol{0}^\circ$
- If the line $\boldsymbol{\ell}$ **is parallel to the** $\boldsymbol{y\text{-}axis}$, then $\boldsymbol{\alpha} = \boldsymbol{90}^\circ$

9. What is the slope or gradient of a line?

Slope or gradient of an inclined path is a measure of its steepness, denoted by m. It is defined as the ratio of rise to run:

$$m = \frac{rise}{run} = \frac{y}{x} = \tan\alpha$$

In analytical geometry, for a non-vertical line with inclination α,

$$m = \tan\alpha$$

10. What are the special cases of slope?

(i) If **ℓ** is **horizontal**, its slope is **zero**.

(ii) If **ℓ** is **vertical**, its slope is **undefined**.

(iii) If **0° < α < 90°**, then **m is positive**.

(iv) If **90° < α < 180°**, then **m is negative**.

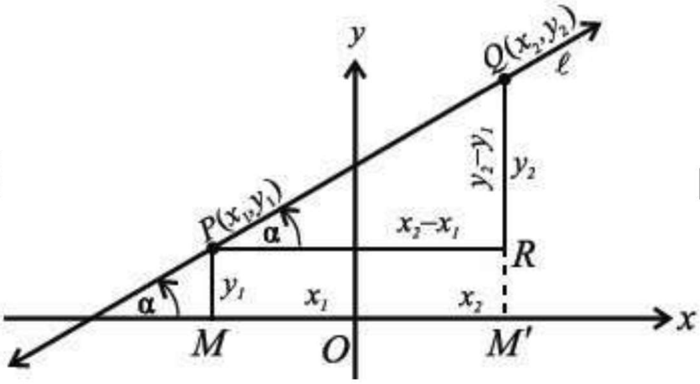
11. What is the formula for the slope of a line passing through two points?

Theorem: If a non-vertical line l with inclination α passes through two points $P(x_1, y_1)$ and $Q(x_2, y_2)$, then the **slope** or **gradient** m of the line is given by:

$$m = \frac{y_2 - y_1}{x_2 - x_1} = \tan\alpha$$

Proof: Let m be the slope of the line l. Draw perpendiculars from the points:

- $\overline{PM}$ from P to the $x\text{-}axis$
- $\overline{QM'}$ from Q to the $x\text{-}axis$
- $\overline{PR}$ from P to $\overline{QM'}$



Now:

- $m\angle RPQ = \alpha$
- $m\overline{PR} = x_2 - x_1$
- $m\overline{QR} = y_2 - y_1$

Using the definition of slope from the right triangle PRQ:

$$m = \tan\alpha = \frac{y_2 - y_1}{x_2 - x_1}$$

Case I: When $0 < \alpha < \frac{\pi}{2}$

In the right triangle PRQ,

$$m = \tan\alpha = \frac{y_2 - y_1}{x_2 - x_1}$$

Case II: When $\frac{\pi}{2} < \alpha < \pi$

In the right triangle PRQ

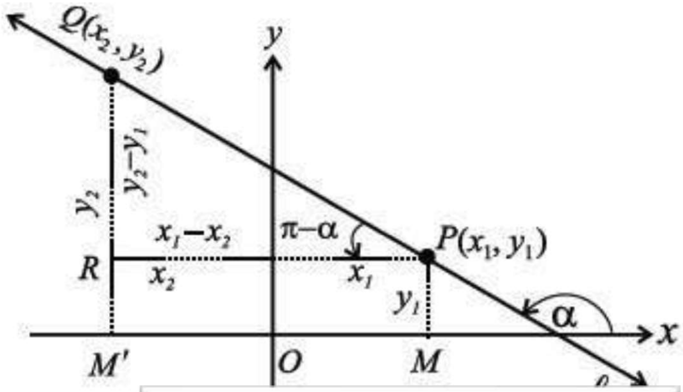
$$\tan(\pi-\alpha)=\frac{y_2-y_1}{x_1-x_2}$$
$$-\tan\alpha=\frac{y_2-y_1}{x_1-x_2}$$
$$\tan\alpha=\frac{y_2-y_1}{x_2-x_1}$$
$$m=\frac{y_2-y_1}{x_2-x_1}$$

Thus, if $P(x_1,y_1)$ and $Q(x_2,y_2)$ are two points on a line, then the slope of $\overleftrightarrow{PQ}$ is given by:

$$m=\frac{y_2-y_1}{x_2-x_1}$$

or

$$m=\frac{y_1-y_2}{x_1-x_2}$$



he symbol:
(i) ∥ stands for "parallel".
(ii) ∦ stands for "not parallel".
(iii) ⊥ stands for "perpendicular"

Theorem 2: Let two lines l_1 and l_2 have slopes m_1 and m_2, respectively:

(i) Lines are **parallel** if and only if $m_1=m_2$

(ii) Lines are **perpendicular** if and only if $m_1=\frac{-1}{m_2}$ or $m_1\cdot m_2=-1$

Note:

(i) $m\neq\frac{y_2-y_1}{x_1-x_2}$ or $m\neq\frac{y_1-y_2}{x_2-x_1}$

(ii) A line l is **horizontal if and only if** $m=0$ $\quad(\because \alpha=0^\circ)$

(iii) A line l is **vertical if and only if** m is not defined $\quad(\because \alpha=90^\circ)$

(iv) If $slope\ of\ \overline{AB}=slope\ of\ \overline{BC}$, then the points A,B, and C are collinear.

12. Show that the points $A(-3,6)$, $B(3,2)$, and $C(6,0)$ are collinear.

$$Slope\ of\ \overline{AB}=\frac{y_2-y_1}{x_2-x_1}$$
$$=\frac{2-6}{3-(-3)}$$
$$=\frac{-4}{6}$$
$$=\frac{-2}{3}$$

and

$$Slope\ of\ \overline{BC}=\frac{y_2-y_1}{x_2-x_1}$$
$$=\frac{0-2}{6-3}$$
$$=\frac{-2}{3}$$
$$\because\ Slope\ of\ \overline{AB}=Slope\ of\ \overline{BC}$$

Hence, the points $\boldsymbol{A},\boldsymbol{B}$, and $\boldsymbol{C}$ **are collinear**.

13. Show that the triangle with vertices $A(1,1)$, $B(4,5)$, and $C(12,-1)$ is a right triangle.

$$Slope\ of\ \overline{AB}=m_1=\frac{y_2-y_1}{x_2-x_1}$$
$$=\frac{5-1}{4-1}$$

$$m_1 = \frac{4}{3}$$

and

$$Slope\ of\ \overline{BC} = m_2 = \frac{y_2 - y_1}{x_2 - x_1}$$
$$= \frac{-1-5}{12-4}$$
$$= \frac{-6}{8}$$
$$m_2 = \frac{-3}{4}$$

Since

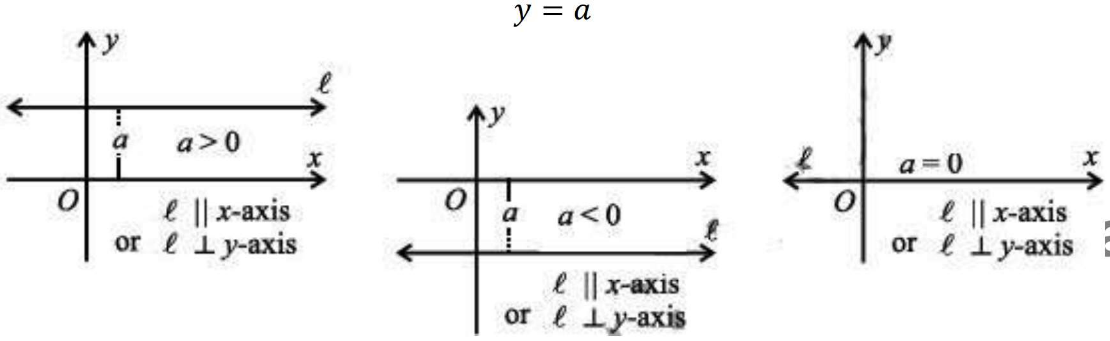
$$m_1 \cdot m_2 = \left(\frac{4}{3}\right)\left(\frac{-3}{4}\right)$$
$$m_1 \cdot m_2 = -1$$

Therefore, $\overline{AB} \perp \overline{BC}$. So ΔABC is a right triangle.

14. What is the equation of a straight line parallel to the x-axis (or perpendicular to the y-axis)?

A line that goes left to right (horizontal) is called a line parallel to the x-axis. It is also said to be perpendicular to the y-axis. The equation of this line is:

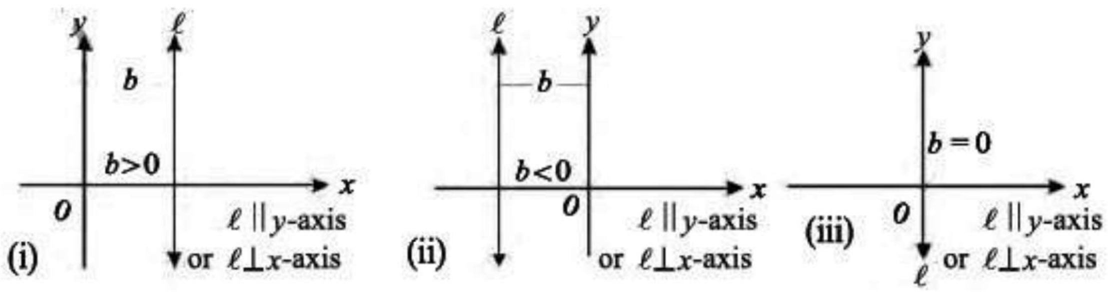
$$y = a$$



14. What is the equation of a straight line parallel to the y-axis (or perpendicular to the x-axis)?

A line that goes up and down (vertical) is called a line parallel to the y-axis. It is also said to be perpendicular to the x-axis. The equation of this line is:

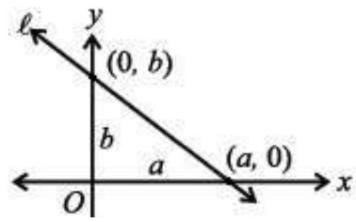
$$x = b$$



15. What are the intercepts of a straight line?

The **x-intercept** of a line is the point where it crosses the **x-axis**. If the line crosses the x-axis at $(a, 0)$, then $\boldsymbol{x\text{-}intercept} = \boldsymbol{a}$.

The **y-intercept** of a line is the point where it crosses the **y-axis**. If the line crosses the y-axis at $(0, b)$, then $\boldsymbol{y\text{-}intercept} = \boldsymbol{b}$.



16. What is the slope-intercept form of a straight line?

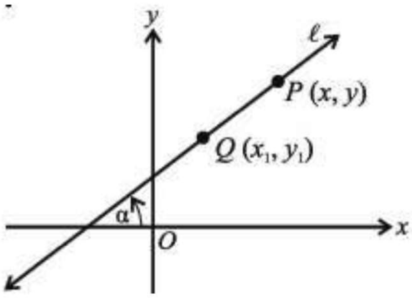
The equation of a non-vertical straight line with slope m and $y\text{-}intercept\ c$ is:

$$y = mx + c$$

Note: If the line passes through the origin, then $c = 0$. So, the equation becomes: $y = mx$

17. What is the Point-Slope Form of the Equation of a Straight Line?

If a non-vertical line with slope m passes through a point $Q(x_1, y_1)$, then its equation is:



$$y - y_1 = m(x - x_1)$$

18. What is the Symmetric Form of the Equation of a Straight Line.

If a line passes through a point (x_1, y_1) and has inclination α, we know from trigonometry:

$$\tan\alpha = \frac{y - y_1}{x - x_1}$$

$$\frac{\sin\alpha}{\cos\alpha} = \frac{y - y_1}{x - x_1}$$

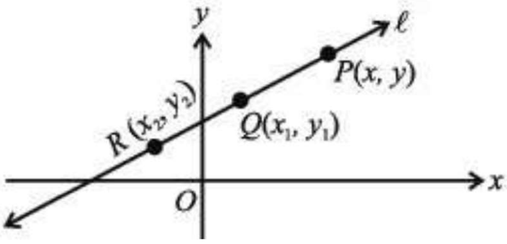
Rewriting:

$$\frac{x - x_1}{\cos\alpha} = \frac{y - y_1}{\sin\alpha} = r \quad (say)$$

This is called the **symmetric form** of the equation of a straight line.

19. What is the equation of a non-vertical straight line passing through two points $Q(x_1, y_1)$and $R(x_1, y_1)$?

The equation of the line passing through two given points is:



$$y - y_1 = \frac{y_2 - y_1}{x_2 - x_1}(x - x_1)$$

or $$y - y_2 = \frac{y_2 - y_1}{x_2 - x_1}(x - x_2)$$

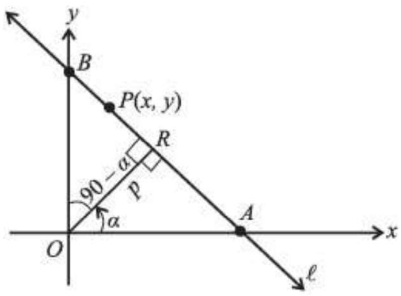
20. What is the equation of a line whose non-zero x-intercept is a and y-intercept is b?

The equation of a line with non-zero x-*intercept* a and y-*intercept* b is:

$$\frac{x}{a} + \frac{y}{b} = 1$$

21. What is the equation of a non-vertical straight line ℓ, where the length of the perpendicular from the origin to ℓ is p, and α is the inclination of this perpendicular?

The equation of the line is:



$$x\cos\alpha + y\sin\alpha = p$$

22. How can the general equation of a line $ax + by + c = 0$ be transformed into standard forms?

The general form of a straight line $ax + by + c = 0$ can be converted into different standard forms as follows:

i. Slope-Intercept Form:

$$ax + by + c = 0$$
$$by = -ax - c$$
$$y = \frac{-ax - c}{b}$$
$$y = -\frac{a}{b}x - \frac{c}{b}$$
$$y = \left(-\frac{a}{b}\right)x + \left(-\frac{c}{b}\right)$$

By comparing with $y = mx + c_1$

$$m = -\frac{a}{b}$$
$$c_1 = -\frac{c}{b}$$

◆ **How do we get the point $\left(-\frac{c}{a},\ 0\right)$?**

From the line equation:

$$ax + by + c = 0$$

Set $y = 0$ to find the **x-intercept**:

$$ax + c = 0 \Rightarrow x = -\frac{c}{a}$$

So, the point is $\left(-\frac{c}{a},\ 0\right)$

ii. Point-Slope Form: We already found slope from general form: $m = -\frac{a}{b}$

Take a known point on the line:

Let $Q\left(-\frac{c}{a}, 0\right)$, Using point-slope form:

$$y - y_1 = m(x - x_1)$$
$$y - 0 = -\frac{a}{b}\left[x - \left(-\frac{c}{a}\right)\right]$$
$$y = -\frac{a}{b}x - \frac{c}{a}$$

◆ **How do we get the point $\left(0,\ -\frac{c}{b}\right)$?**

Set $x = 0$ to find the **y-intercept**:

$$by + c = 0 \Rightarrow y = -\frac{c}{b}$$

So, the point is $\left(0,\ -\frac{c}{b}\right)$

iii. Symmetric Form: We know from slope: $m = \tan\alpha = -\frac{a}{b}$

Use trigonometric identities: $\sin\alpha = \frac{a}{\pm\sqrt{a^2+b^2}}$, $\cos\alpha \frac{b}{\pm\sqrt{a^2+b^2}}$

Using point $Q\left(-\frac{c}{a}, 0\right)$ in $\frac{x-x_1}{\cos\alpha} = \frac{y-y_1}{\sin\alpha} = r \quad (say)$

$$\frac{x + \frac{c}{a}}{b/\pm\sqrt{a^2+b^2}} = \frac{y - 0}{a/\pm\sqrt{a^2+b^2}} = r \quad (say)$$

iv. Two-Point Form: Choose two points on the line: $A\left(-\frac{c}{a}, 0\right)$ and $B\left(0, -\frac{c}{b}\right)$

Use two-point form:

$$y - y_1 = \frac{y_2 - y_1}{x_2 - x_1}(x - x_1)$$
$$y = \frac{y_2 - y_1}{x_2 - x_1}(x - x_1) + y_1$$
$$y = \frac{-\frac{c}{b} - 0}{0 + \frac{c}{a}}\left(x + \frac{c}{a}\right) + 0$$
$$y = \frac{-\frac{c}{b}}{\frac{c}{a}}\left(x + \frac{c}{a}\right)$$
$$y = \frac{-ca}{cb}\left(x + \frac{c}{a}\right)$$
$$y = \frac{-a}{b}\left(x + \frac{c}{a}\right)$$
$$y = \frac{-a}{b}x + \frac{-ac}{ba}$$

$$y = \left(-\frac{a}{b}\right)x - \frac{c}{b}$$

v. Intercept Form:

$$ax + by + c = 0$$
$$ax + by = -c$$
$$\frac{ax}{-c} + \frac{by}{-c} = \frac{-c}{-c}$$
$$\frac{x}{{}^{-c}/_{a}} + \frac{y}{{}^{-c}/_{b}} = 1$$

Where $\boldsymbol{x\text{-}intercept} = {}^{-c}/_{a}$ and $\boldsymbol{y\text{-}intercept} = {}^{-c}/_{b}$

vi. Normal Form:

$$ax + by + c = 0$$
$$\frac{ax}{\pm\sqrt{a^2+b^2}} + \frac{by}{\pm\sqrt{a^2+b^2}} = \frac{-c}{\pm\sqrt{a^2+b^2}}$$
$$x\left(\frac{a}{\pm\sqrt{a^2+b^2}}\right) + y\left(\frac{b}{\pm\sqrt{a^2+b^2}}\right) = \frac{-c}{\pm\sqrt{a^2+b^2}}$$
$$x\cos\alpha + y\sin\alpha = p$$

Where, $\cos\alpha = \frac{a}{\pm\sqrt{a^2+b^2}}$, $\sin\alpha \frac{b}{\pm\sqrt{a^2+b^2}}$ and $p = \frac{-c}{\pm\sqrt{a^2+b^2}}$