Unit 9 Similar Figures

1. What are similar polygons?

Similar figures have the same shape but not necessarily the same size. Two polygons are similar if:

(i) Their corresponding $angles$ are equal, and

(ii) Their corresponding $sides\ are\ proportional$ ($i.e.$, the ratios of the lengths of corresponding sides are equal).

Note: Proportionality of sides means one side is k times of its corresponding side.

2. Define polygon.

Three or more than three-sided closed figure is called polygon.

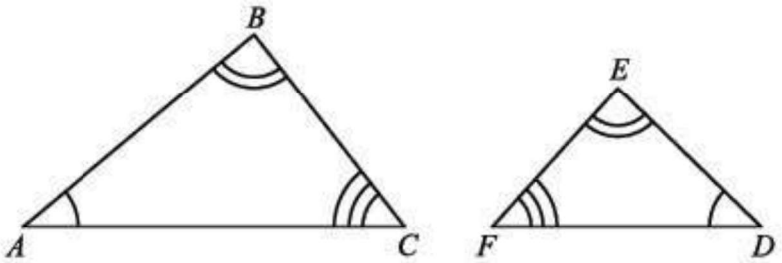
3. How can we identify similar triangles?

If two angles in one triangle are congruent to two corresponding angles in another triangle, the third angle in each triangle must also be congruent. Since the angles are the same, the triangles are similar. The similarity **symbol** is $\sim$.

For example, in the correspondence of triangles ABC and DEF, if

$$m\angle A = m\angle D$$
$$m\angle B = m\angle E$$
$$m\angle C = m\angle F$$

then triangle $\Delta ABC \sim \Delta DEF$.



4. What is the rule for finding the ratio of areas of similar figures?

The ratio of areas of two similar figures is equal to the square of the ratio of their corresponding lengths.

$$\frac{A_1}{A_2} = \left(\frac{l_1}{l_2}\right)^2$$

Where:

- A_1, A_2 = Areas of the two figures
- l_1, l_2 = Their corresponding lengths

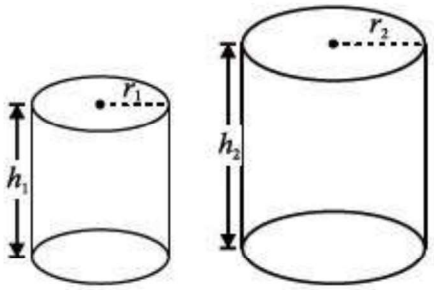
5. What are similar solids? Explain with an example.

Two solids are said to be similar if they have the same shape but possibly different sizes.

For example, two cylinders are similar if:

$$\frac{r_1}{r_2} = \frac{h_1}{h_2}$$

where r_1, r_2 are the radii and h_1, h_2 are the heights of the two cylinders.



6. What is the volume ratio of two similar solids?

For two similar solids, the ratio of their volumes is equal to the cube of the ratio of their corresponding lengths:

$$\frac{V_1}{V_2} = \left(\frac{l_1}{l_2}\right)^3$$

7. What is the relationship between mass and volume of similar solids?

For solids made of the same material, mass is directly proportional to volume:

$$\frac{w_1}{w_2} = \frac{V_1}{V_2}$$

So

$$\frac{w_1}{w_2} = \left(\frac{l_1}{l_2}\right)^3$$

where w_1 and w_2 are the masses of the similar solids.

6. Define regular polygon.

A regular polygon has all sides and all angles equal. Some of the common regular polygons are equilateral triangles, squares, regular pentagons, regular hexagons, etc.

7. What is the sum of interior angles of a regular polygon?

The formula for sum of interior angles of an $n\text{-}sided$ polygon is:

$$(n-2) \times 180^\circ$$

8. How do you calculate the size of each interior angle of a regular polygon?

For a regular $n\text{-}sided$ polygon,

$$Size\ of\ each\ Interior\ Angle = \frac{(n-2) \times 180^\circ}{n}$$

For example, for a regular hexagon ($n = 6$),

$$Each\ interior\ angle = \frac{(6-2) \times 180^\circ}{6}$$
$$= \frac{720^\circ}{6}$$
$$= 120^\circ$$

9. What is the sum and formula of the exterior angle of a regular polygon?

The sum of all exterior angles of any polygon is always 360°, regardless of the number of sides.

The exterior angle of a regular $n\text{-}sided$ polygon is:

$$Exterior\ Angle = \frac{360^\circ}{n}$$

Note: The interior and exterior angles are supplementary at a vertex. That is,

$$Interior\ Angle + Exterior\ Angle = 180^\circ$$

10. How do you find the number of diagonals in a regular polygon?

The total number of diagonals in a regular polygon with $n\text{-}sided$ is $\frac{n(n-3)}{2}$.

11. What symmetry does a regular polygon have?

A regular $n\text{-}sided$ polygon has rotational symmetry and reflexive (line) symmetry, both of order n.

For example, a regular hexagon has six lines of symmetry and rotational symmetry of order 6. It can be rotated by $\frac{360^\circ}{n}$ and still look the same.

12. What are the geometrical properties of a triangle?

A **triangle** is a polygon with three sides and three angles.

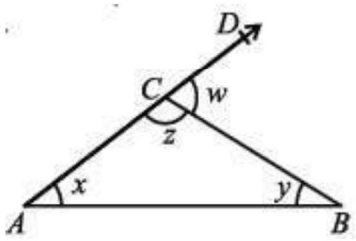
Triangles come in various types based on side length and angle measure.

(i) **Angle sum:** The sum of the interior angles in any triangle is always 180°.

(ii) In an **equilateral triangle**, all sides are equal and each angle is 60°. It has **three lines** of symmetry and rotational symmetry of $order\ 3$.

(iii) In an **isosceles triangle**, two sides are equal, and the angles opposite to the equal sides are also equal. It has **one line of symmetry**.

13. What is the exterior angle property of a triangle?



The measure of an exterior angle in a triangle is equal to the sum of the measures of two opposite interior angles. In ΔABC,

$$m\angle A + m\angle B = m\angle BCD$$
$$x + y = w$$

14. What are the geometrical properties of a parallelogram?

A **parallelogram** is a $quadrilateral$ whose opposite sides are parallel and equal in length and opposite angles are equal. Its adjacent angles are supplementary.

The diagonals of a parallelogram bisect each other (they cross each other at the midpoint). They are not equal in length.

15. What are the properties of rectangle, rhombus, and square?

Rectangle: All angles are **90°** and diagonals are equal.

Rhombus: All sides are **equal**, and diagonals bisect each other at right angles.

Square: All sides are **equal**, all angles are **90°**, and diagonals are equal and bisect each other at right angles.

16. Define tessellation.

A tessellation is a pattern of shapes that fit together perfectly without any gaps or overlaps to completely cover a flat surface.

17. Which regular polygons can tessellate the plane on their own?

Only **three regular polygons** can tessellate the plane on their own:

(i) **Equilateral triangles** (each angle is 60°, six triangles meet at a point $= 360^\circ$)

(ii) **Squares** (each angle is 90°, four squares meet at a point = 360°)

(iii) **Regular hexagons** (each angle is 120°, three hexagons meet at a point $= 360^\circ$)

18. Why can't regular pentagons tessellate the plane?

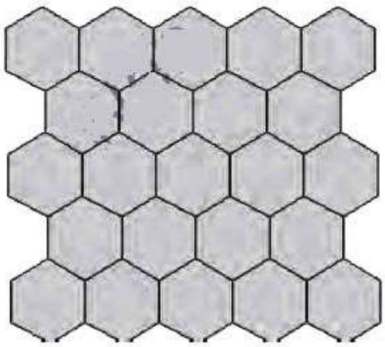
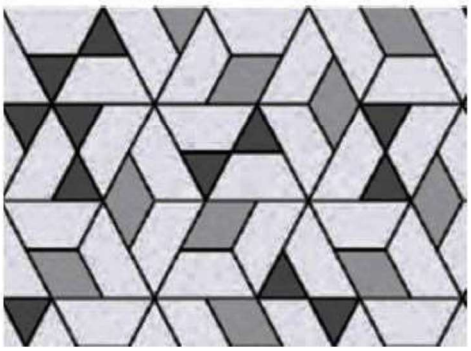
Regular pentagons and other polygons with angles that don't add up to 360° at each vertex cannot form gap-free patterns, so tessellation is not possible.

19. How are polygons used in real life?

Polygons are used in **video games and animations** to build characters and scenes. In **science**, they appear in **molecular shapes**, **honeycomb patterns** (hexagonal), and even in the design of **telescope mirrors**.

20. What is the difference between regular and irregular tessellation?

Regular Tessellation	Irregular Tessellation
A tessellation is a pattern of shapes that fit together perfectly without any gaps or overlaps to completely cover a flat surface.	A pattern made using different shapes: regular polygons, irregular polygons, or both.
Uses same regular polygons only.	Uses different or irregular polygons, or a mix of both.
Tessellations using: - Equilateral triangles - Squares - Regular hexagons	Tessellations using: - Squares and triangles - Irregular quadrilaterals - Hexagons and irregular pentagons

Regular Tessellation	Irregular Tessellation
	

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