Numerical Problems

Important formulas:

- **Intensity Level of Sound**

$$Intensity\ level = 10 \log \frac{I}{I_o}$$

- **Speed of wave** $v = f\lambda$
- **Time period** $T = \frac{1}{f}$
- **Frequency** $f = \frac{n}{t}$
- **Speed** $v = \frac{d}{t}$
- **Distance** $S = vt$

11.1 A normal conversation involves sound intensities of about $3.0 \times 10^{-6}\ Wm^{-2}$. What is the decibel level for this intensity? What is the intensity of the sound for $100\ dB$? *(ALP)*

Given Data

$$Intensitieis\ of\ sound = I = 3.0 \times 10^{-6}\ Wm^{-2}$$
$$Int.of\ faintest\ sound = I_o = 10^{-12}\ Wm^{-2}$$
$$Sound\ level\ = L - L_o = 100\ dB$$

To Find

$$Intensity\ level = L - L_o = ?$$
$$Intensity\ of\ sound = I = ?$$

Solution

By using formula of intensity level of sound

$$L - L_o = 10 \log \frac{I}{I_o}$$
$$L - L_o = 10 \log \frac{3.0 \times 10^{-6}}{10^{-12}}$$
$$L - L_o = 10 \log 3.0 \times 10^{-6+12}$$
$$L - L_o = 10 \log 3.0 \times 10^{6}$$
$$L - L_o = 10 \times 6.477$$
$$\boldsymbol{L - L_o = 64.77\ dB}$$

Now, again by using formula of intensity level of sound

$$L - L_o = 10 \log \frac{I}{I_o}$$
$$100 = 10 \log \frac{I}{10^{-12}}$$
$$\frac{100}{10} = \log \frac{I}{10^{-12}}$$
$$10 = \log \frac{I}{10^{-12}}$$
$$Antilog(10) = Antilog\left(\log \frac{I}{10^{-12}}\right)$$
$$1 \times 10^{10} = \frac{I}{10^{-12}}$$
$$1 \times 10^{10} \times 10^{-12} = I$$
$$1 \times 10^{10-12} = I$$
$$1 \times 10^{-2} = I$$
$$0.01 = I$$
$$\boldsymbol{I = 0.01\ Wm^{-2}}$$

11.2 If at Anarkali Bazar Lahore, intensity level of sound is $80\ dB$, what will be the intensity of sound there? *(ALP)*

Given Data

$$Sound\ level\ = L - L_o = 80\ dB$$
$$Intensity\ of\ faintest\ sound = I_o = 10^{-12}\ Wm^{-2}$$

To Find

$$Intensity\ of\ sound = I = ?$$

Solution

By using formula of intensity level of sound

$$L - L_o = 10 \log \frac{I}{I_o}$$
$$80 = 10 \log \frac{I}{10^{-12}}$$
$$\frac{80}{10} = \log \frac{I}{10^{-12}}$$
$$8 = \log \frac{I}{10^{-12}}$$
$$Antilog(8) = Antilog\left(\log \frac{I}{10^{-12}}\right)$$
$$1 \times 10^{8} = \frac{I}{10^{-12}}$$
$$1 \times 10^{8} \times 10^{-12} = I$$
$$1 \times 10^{8-12} = I$$
$$1 \times 10^{-4} = I$$
$$\boldsymbol{I = 10^{-4}\ Wm^{-2}}$$

11.3 At a particular temperature, the speed of sound in air is $330\ ms^{-1}$. If the wavelength of a note is $5\ cm$, calculate the frequency of the sound wave. Is this frequency in the audible range of the human ear? *(ALP)*

Given Data

$$Speed\ of\ sound = v = 330\ ms^{-1}$$
$$Wavelength = \lambda = 5\ cm$$
$$\lambda = 5 \times 10^{-2} m$$
$$\lambda = 0.05\ m$$

To Find

$$Frequency = f = ?$$

Solution

By using formula of speed of wave

$$v = f\lambda$$
$$330 = (f)(0.05)$$
$$\frac{330}{0.05} = f$$
$$6600 = f$$
$$f = 6600\ Hz$$
$$\boldsymbol{f = 6.6 \times 10^{3}\ Hz}$$

Yes, the frequency lies in the audible range of human ear.

11.4 A doctor counts 72 heartbeats in 1 min. Calculate the frequency and period of the heartbeats. *(ALP)*

Given Data

$$Number\ of\ heartbeats = n = 72$$
$$Time = t = 1\ min$$
$$t = 60\ s$$

To Find

$$requency = f = ?$$
$$Time\ period = T = ?$$

Solution

For frequency, we use

$$f = \frac{n}{t}$$
$$f = \frac{72}{60}$$
$$\boldsymbol{f = 1.2\ Hz}$$

For time period, we use

$$T = \frac{1}{f}$$
$$T = \frac{1}{1.2}$$
$$\boldsymbol{T = 0.833\ s}$$

11.5 A marine survey ship sends a sound wave straight to the seabed. It receives an echo $1.5\ s$ later. The speed of sound in seawater is $1500\ ms^{-1}$. Find the depth of the sea at this position.

Given Data

$$Time\ taken = t = 1.5\ s$$
$$Speed = v = 1500\ ms^{-1}$$

To Find

$$Depth\ of\ seabed = h = ?$$

Solution

By using formula of distance

$$S = vt$$
$$S = (1500)(1.5)$$
$$S = 2250\ m$$

The echo distance must be half, so

$$h = \frac{S}{2}$$
$$h = \frac{2250}{2}$$
$$\boldsymbol{h = 1150\ m}$$

11. 6 A student clapped his hands near a cliff and heard the echo after $5\ s$. What is the distance of the cliff from the student if the speed of the sound is taken as $346\ ms^{-1}$? *(ALP)*

Given Data

$$Time\ taken = t = 5\ s$$
$$Speed = v = 346\ ms^{-1}$$

To Find

$$Distance\ of\ cliff = h = ?$$

Solution

By using formula of distance

$$S = vt$$
$$S = (346)(5)$$
$$S = 1730\ m$$

The echo distance must be half, so

$$h = \frac{S}{2}$$
$$h = \frac{1730}{2}$$
$$\boldsymbol{h = 865\ m}$$

11.7 A ship sends out ultrasound that returns from the seabed and is detected after $3.42\ s$. If the speed of ultrasound through seawater is $1531\ ms^{-1}$, what is the distance of the seabed from the ship?

Given Data

$$Time\ taken = t = 3.42\ s$$
$$Speed = v = 1531\ ms^{-1}$$

To Find

$$Distance\ of\ seabed = h = ?$$

Solution

By using formula of distance

$$S = vt$$
$$S = (1531)(3.42)$$
$$S = 5236.02\ m$$

The echo distance must be half, so

$$h = \frac{S}{2}$$
$$h = \frac{5236.02}{2}$$
$$\boldsymbol{h = 2618.01\ m}$$

11.8 The highest frequency sound humans can hear is about $20,000\ Hz$. What is the wavelength of sound in air at this frequency at a temperature of $20℃$? What is the wavelength of the lowest sounds we can hear of about $20\ Hz$? Assume the speed of sound in air at 20 OC is $343\ ms^{-1}$.

Given Data

$$Highest\ frequency = f_1 = 20000\ Hz$$
$$Lowest\ frequency = f_2 = 20\ Hz$$
$$Speed\ of\ sound = v = 343\ ms^{-1}$$

To Find

$$Wavelength\ at\ f_1 = \lambda_1 = ?$$
$$Wavelength\ at\ f_2 = \lambda_2 = ?$$

Solution

By using formula of speed of wave

$$v = f_1\lambda_1$$
$$343 = (20000)(\lambda_1)$$
$$\frac{343}{20000} = \lambda_1$$
$$0.01715 = \lambda_1$$
$$\lambda_1 = 0.01715\ m$$
$$\boldsymbol{\lambda_1 = 1.715 \times 10^{-2}\ m}$$

Now again by using formula of speed of wave

$$v = f_2\lambda_2$$
$$343 = (20)(\lambda_2)$$
$$\frac{343}{20} = \lambda_2$$
$$17.15 = \lambda_2$$
$$\lambda_2 = 17.15\ m$$
$$\boldsymbol{\lambda_2 = 1.715 \times 10^{1}\ m}$$

11.9 A sound wave has a frequency of $2\ kHz$ and wavelength $35\ cm$. How long will it take to travel $1.5\ km$? *(ALP)*

Given Data

$$Frequency = f = 2\ kHz$$
$$Frequency = f = 2 \times 10^3\ Hz$$
$$Wavelength = \lambda = 35\ cm$$
$$\lambda = 35 \times 10^{-2} m$$
$$\lambda = 0.35\ m$$
$$Distance = S = 1.5\ km$$
$$S = 1.5 \times 10^3\ m$$

To Find

$$Time = t = ?$$

Solution

First, we find speed of wave

$$v = f\lambda$$
$$v = (2 \times 10^3)(0.35)$$
$$v = 700\ ms^{-1}$$

Now by using formula of distance

$$S = vt$$
$$1.5 \times 10^3 = (700)(t)$$
$$\frac{1.5 \times 10^3}{700} = t$$
$$2.14 = t$$
$$\boldsymbol{t = 2.14\ s}$$

Examples

11. 1 Calculate the intensity levels of the (a) faintest audible sound (b) rustling of leaves. ***(ALP)***

Solution

(a) Intensity level of faintest audible sound can be calculated by substituting $I = I_o = 10^{-12}\ Wm^{-2}$. Therefore, by using formula of intensity level of sound

$$L - L_o = 10 \log \frac{I}{I_o}$$
$$L - L_o = 10 \log \frac{I_o}{I_o} \qquad \because I = I_o$$
$$L - L_o = 10 \log 1$$
$$L - L_o = (10)(0)$$
$$\boldsymbol{L - L_o = 0\ dB}$$

(b) As intensity of the rustle of leave is $I = 10^{-11}\ Wm^{-2}$. Therefore, by using formula of intensity level of sound

$$L - L_o = 10 \log \frac{I}{I_o}$$
$$L - L_o = 10 \log \frac{10^{-11}}{10^{-12}}$$
$$L - L_o = 10 \log 10^{-11+12}$$
$$L - L_o = 10 \log 10$$
$$L - L_o = (10)(1)$$
$$\boldsymbol{L - L_o = 10\ dB}$$

11.2 Calculate the frequency of a sound wave of speed $340\ ms^{-1}$ and wavelength 0.5 m. ***(ALP)***

Given Data

$$Speed\ of\ sound = v = 340\ ms^{-1}$$
$$Wavelength = \lambda = 0.5\ m$$

To Find

$$Frequency = f = ?$$

Solution

By using formula of speed of wave

$$v = f\lambda$$
$$340 = (f)(0.5)$$
$$\frac{340}{0.5} = f$$
$$680 = f$$
$$\boldsymbol{f = 680\ Hz}$$

11. 3 Flash of lightning is seen $1.5\ seconds$ earlier than the thunder. How far away is the cloud in which the flash has occurred? (speed of sound $= 332\ ms^{-1}$).

Given Data

$$Time\ taken = t = 1.5\ s$$
$$Speed = v = 332\ ms^{-1}$$

To Find

$$Distance = S = ?$$

Solution

By using formula of distance

$$S = vt$$
$$S = (332)(1.5)$$
$$\boldsymbol{S = 498\ m}$$