Numerical Problems

Important formulas

- **No. of half life**

$$No. of\ half\ life = \frac{time}{half\ life}$$

$$n = \frac{t}{T_{1/2}}$$

- **Remaining quantity**

$$Remaining\ quantity = \frac{1}{2^t} \times Orignal\ quantity$$

$$N = \frac{1}{2^n} \times N_o$$

18.1 The half-life of $^{16}_{7}N$ is $7.3s$. A sample of this nuclide of nitrogen is observer for $29.2s$. Calculate the fraction of the original isotope remaining after this time. *(ALP)*

Given Data

$$Half\ life = T_{1/2} = 7.3s$$

$$Time = t = 29.2s$$

$$Orignal\ quantity = N_o$$

To Find

$$Remaining\ quantity = N = ?$$

Solution

$$No. of\ half\ life = \frac{time}{half\ life}$$

$$= \frac{t}{T_{1/2}}$$

$$n = \frac{29.2}{7.3}$$

$$n = 4$$

Now

$$Remaining\ quantity\ of\ Nitroge = \frac{1}{2^n} \times Orignal\ quantity$$

$$N = \frac{1}{2^n} \times N_o$$

$$N = \frac{1}{2^4} \times N_o$$

$$\boldsymbol{N = \frac{1}{16} \times N_o}$$

So, after $\boldsymbol{29.2s}$, $\frac{1}{16^{th}}$ part of original sample is left.

18.2 Cobalt-60 is a radioactive element with half-life of 5.25 years. What fraction of the original sample will be left after 26 years? *(ALP)*

Given Data

$$Half\ life = T_{1/2} = 5.25\ years$$

$$Time = t = 26\ years$$

$$Orignal\ quantity = N_o$$

To Find

$$Remaining\ quantity = N = ?$$

Solution

$$No. of\ half\ life = \frac{time}{half\ life}$$

$$n = \frac{t}{T_{1/2}}$$

$$n = \frac{26}{5.25}$$

$$n = 4.95$$

$$n \approx 5$$

Now

$$Remaining\ quantity = \frac{1}{2^n} \times Orignal\ quantity$$

$$N = \frac{1}{2^n} \times N_o$$

$$N = \frac{1}{2^5} \times N_o$$

$$\boldsymbol{N = \frac{1}{32} \times N_o}$$

So, after $\boldsymbol{26\ years}$, $\frac{1}{32}$ part of original sample is left.

18.3 Carbon-14 has half-life of 5730 years. How long will it take for the quantity of carbon-14 in a sample to drop to one eight of initial quantity? *(ALP)*

Given Data

$$Half\ life = T_{1/2} = 5730\ years$$

$$Initial\ quantity = N_o$$

$$Remaining\ quantit = N = \frac{1}{8} \times N_o$$

To Find

$$Time = t = ?$$

Solution

$$Remaining\ quantity = \frac{1}{2^n} \times Orignal\ quantity$$

$$N = \frac{1}{2^n} \times N_o$$

$$\frac{1}{8} \times N_o = \frac{1}{2^n} \times N_o$$

$$\frac{1}{8} = \frac{1}{2^n}$$

$$2^n = 8$$

$$2^n = 2^3$$

$$\Rightarrow \quad n = 3$$

Now

$$No. of\ half\ life = \frac{time}{half\ life}$$

$$n = \frac{t}{T_{1/2}}$$

$$3 = \frac{t}{5730\ years}$$

$$3 \times 5730\ years = t$$

$$t = 17190\ years$$

$$\boldsymbol{t = 1.719 \times 10^4\ years}$$

18.4 Tcehnetium-99 m is a radioactive element and is used to diagnose brain, thyroid, liver and kidney diseases. This element has half-life of 6 hours. If there is 200 mg of this technetium present, how much will be left in 36 hours.

Given Data

$$Half\ life = T_{1/2} = 6\ hours$$

$$Time = t = 36\ hours$$

$$Orignal\ quantity = N_o = 200mg$$

To Find

$$Remaining\ quantity = N = ?$$

Solution

$$No. of\ half\ life = \frac{time}{half\ life}$$
$$n = \frac{t}{T_{1/2}}$$
$$n = \frac{36}{6}$$
$$n = 6$$

Now

$$Remaining\ quantity = \frac{1}{2^n} \times orignal\ quantity$$
$$N = \frac{1}{2^n} \times N_o$$
$$N = \frac{1}{2^6} \times 200\ mg$$
$$N = \frac{1}{64} \times 200mg$$
$$N = \frac{200mg}{64}$$
$$\boldsymbol{N = 3.125\ mg}$$

18.5 Half-life of a radioactive element is 10 minutes. If the initial count rate is 368 counts per minute, find the time for which count rates reaches 23 counts per minute. ***(ALP)***

Given Data

$$Half\ life = T_{1/2} = 10\ minutes$$
$$Initial\ count\ rate = 368\ per\ minute$$

To Find

$$time = t = ?$$

Solution

The initial count rate is 368 counts per minute, therefore

$$368 \xrightarrow{10\ min.} 184 \xrightarrow{10\ min.} 92 \xrightarrow{10\ min.} 46 \xrightarrow{10\ min.} 23$$

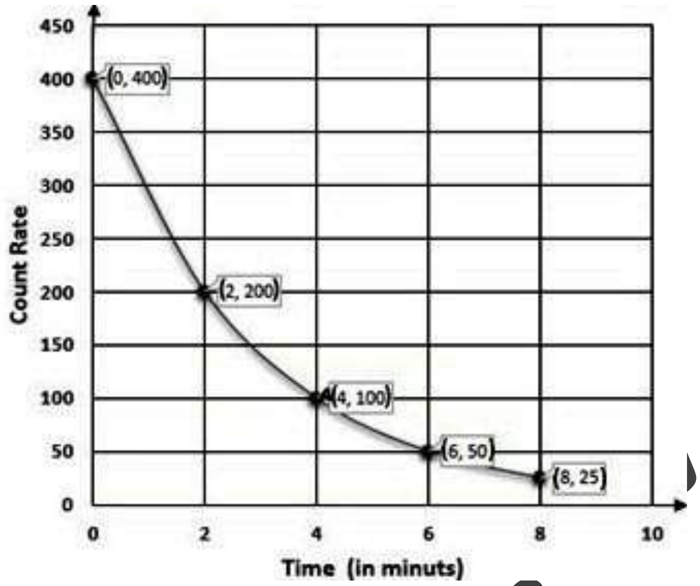
As clear, it takes 4 half-lives for count rate decrease from 368 to 23 per minutes, so

$$No. of\ half\ life = \frac{time}{half\ life}$$
$$n = \frac{t}{T_{1/2}}$$
$$4 = \frac{t}{10\ min.}$$
$$4 \times 10\ min. = t$$
$$\boldsymbol{t = 40\ min.}$$

18.6 In an experiment to measure the half-life of a radioactive element, the following results were obtained:

Count rate / minute	400	200	100	50	25
Time (in minutes)	0	2	4	6	8

Plot a graph between the count rate and time in minutes. Measure the value for the half-life of the element from the graph.



Solution

Graph between time and count rate:

After two successive half-lives, let 'A' point which is $(n = 2)$.

$$No. of\ half\ life = \frac{time}{half\ life}$$
$$n = \frac{t}{T_{1/2}}$$
$$2 = \frac{4\ min.}{T_{1/2}}$$
$$T_{1/2} = \frac{4\ min.}{2}$$
$$\boldsymbol{T_{1/2} = 2\ min.}$$

18.7 A sample of certain radioactive element has a half-life of 1500 years. If it has an activity of 32000 counts per hour at the present time, then plot a graph of the activity of this sample over the period in which it will reduce to 1/16 of its present value. ***(ALP)***

Given Data

$$Half - life = T_{1/2}$$
$$= 1500\ years$$
$$Initial\ count\ per\ hour = N_o = 32000$$
$$Remaining\ \frac{1}{16}\ of\ initial\ count\ rate = N$$
$$= \frac{1}{16} \times 32000$$

To Find

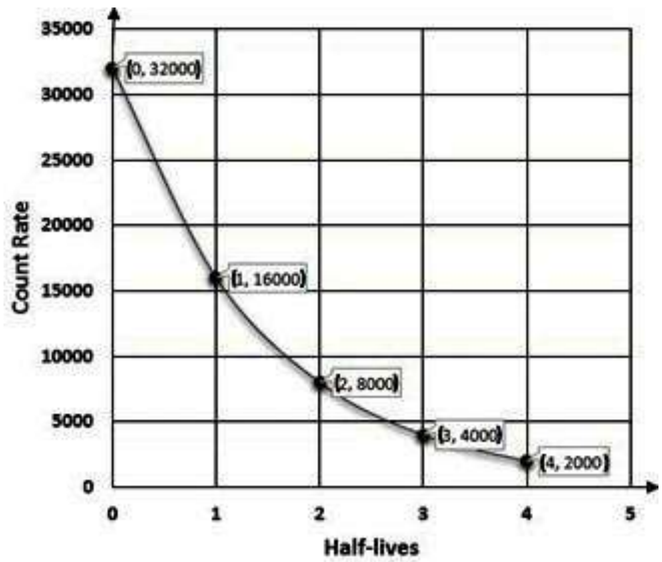
$$Time = t = ?$$

Solution

$$Remaining\ quantity = \frac{1}{2^n} \times Orignal\ quantity$$
$$N = \frac{1}{2^n} \times N_o$$
$$\frac{1}{16} \times 32000 = \frac{1}{2^n} \times 32000$$
$$\frac{1}{16} = \frac{1}{2^n}$$
$$2^n = 16$$
$$2^n = 2^4$$
$$\Rightarrow \boldsymbol{n = 4}$$

Count Rate	32000	16000	8000	4000	2000

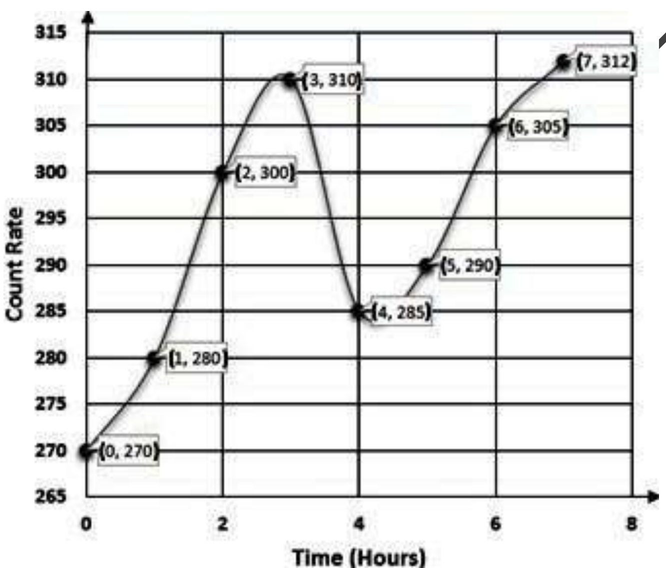
Half-lives	0	1	2	3	4



18.8 Half-life of a radioactive element was found to be 4000 years. The count rates per minute for 8 successive hours were found to be $270, 280, 300, 310, 285, 290, 305, 312$. What does the variation in count rates show? Plot a graph between the count rates and time in hours. Why the graph is a straight line rather than an exponential?

Solution

Variation in count rate shows the random nature of radioactive decay, graph is almost horizontal line rather than exponential curve which is due to long half-life as compared to period of 8 hours.



18.9 Ashes from a campfire deep in a cave show corbon-14 activity of only one-eighth the activity of fresh wood. How long ago was that campfire made? *(ALP)*

Given Data

$$Orignal\ quantity\ = N_o$$

$$Remaining\ quantity = N = \frac{1}{8} \times N_o$$

$$Half-life\ of\ corbon-14 = T_{1/2} = 5730\ years$$

To Find

$$Time = t = ?$$

Solution

$$Remaining\ quantity = \frac{1}{2^n} \times Orignal\ quantity$$

$$N = \frac{1}{2^n} \times N_o$$

$$\frac{1}{8} \times N_o = \frac{1}{2^n} \times N_o$$

$$\frac{1}{8} = \frac{1}{2^n}$$

$$2^n = 8$$

$$2^n = 2^3$$

$$\Rightarrow \quad n = 3$$

Now

$$No. of\ half\ life = \frac{time}{half\ life}$$

$$n = \frac{t}{T_{1/2}}$$

$$3 = \frac{t}{5730\ years}$$

$$3 \times 5730\ years = t$$

$$\boldsymbol{t = 17190\ years}$$

Examples

18.1 Find the number of protons and neutrons in nuclide defined by $^{13}_{6}X$. *(ALP)*

Given Data

$$Atomic\ no. = Z = 6$$

$$mass\ no. = A = 13$$

To Find

$$No. of\ protons = Z = ?$$

$$No. of\ neutrons = n = ?$$

Solution

$$\boldsymbol{Z = no. of\ protons = 6}$$

Also

$$Atomic\ mass = no. of\ protons + no.f\ neutrons$$

$$A = Z + n$$

$$13 = 6 + n$$

$$13 - 6 = n$$

$$7 = n$$

$$\boldsymbol{n = 7}$$

18.2 The activity of a sample of radioactive bismuth decrease to one-eight of its original activity in 15 days. Calculate the half life of the sample. *(ALP)*

Given Data

$$Orignal\ quantity\ = N_o$$

$$Remaining\ quantity = N = \frac{1}{8} \times N_o$$

$$Time = t = 15\ days$$

To Find

$$Half-life\ of\ bismuth = T_{1/2} = ?$$

Solution

$$Remaining\ quantity = \frac{1}{2^n} \times Orignal\ quantity$$

$$N = \frac{1}{2^n} \times N_o$$

$$\frac{1}{8} \times N_o = \frac{1}{2^n} \times N_o$$

$$\frac{1}{8} = \frac{1}{2^n}$$

$$2^n = 8$$

$$2^n = 2^3$$

$$\Rightarrow \quad n = 3$$

Now

$$No.\,of\ half\ life = \frac{time}{half\ life}$$
$$n = \frac{t}{T_{1/2}}$$
$$3 = \frac{15\ days}{T_{1/2}}$$
$$T_{1/2} = \frac{15\ days}{3}$$
$$T_{1/2} = \mathbf{5\ days}$$

18.3 A radioactive element has a half-life of 40 minutes. The initial count rate was 1000 per minute. How long it take for the count rate to drope to (a) 250 per minutes (b) 125 per minutes (c) plot a graph of the radioactive decay of the element.

Give Data

$$half\ life = T_{1/2} = 40\ minutes$$
$$initial\ count\ rate = 1000\ per\ minute$$

To Find

$$time = t = ?$$

Solution

The initial count rate is 1000 counts per minute, therefore

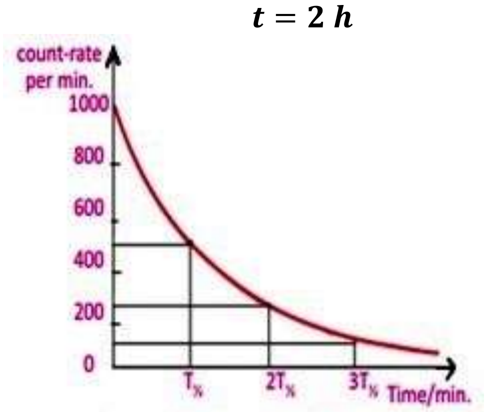
$$1000 \xrightarrow{40\ min.} 500 \xrightarrow{40\ min.} 250 \xrightarrow{40\ min.} 125$$

(a) As clear, it takes 2 half-lives for count rate decrease from 1000 to 250 per minutes, so

$$No.\,of\ half\ life = \frac{time}{half\ life}$$
$$n = \frac{t}{T_{1/2}}$$
$$2 = \frac{t}{40\ min.}$$
$$2 \times 40\ min. = t$$
$$\boldsymbol{t = 80\ min.}$$

(b) As clear, it take 3 half-lives for count rate decrease from 1000 to 125 per minutes, so

$$No.\,of\ half\ life = \frac{time}{half\ life}$$
$$n = \frac{t}{T_{1/2}}$$
$$3 = \frac{t}{40\ min.}$$
$$3 \times 40\ min. = t$$
$$t = 120\ min.$$
$$\boldsymbol{t = 2\ h}$$



18.4 $C-14$: $C-12$ ratio in a fossil bone is found to be $\frac{1}{4^{th}}$ that of the ratio in the bone of a living animal. The half-life of $C-14$ is 5730 years what is approximate age of the fossil? *(ALP)*

Given Data

$$Half\ life = T_{1/2} = 5730\ years$$
$$\frac{N}{N_\circ} = \frac{1}{4}$$

To Find

$$Time = t = ?$$

Solution

$$Remaining\ quantity = \frac{1}{2^n} \times Orignal\ quantity$$
$$N = \frac{1}{2^n} \times N_o$$
$$\frac{N}{N_o} = \frac{1}{2^n}$$
$$\frac{1}{4} = \frac{1}{2^n}$$
$$\frac{1}{2^2} = \frac{1}{2^n}$$
$$\Rightarrow n = 2$$

Now

$$No.\,of\ half\ life = \frac{time}{half\ life}$$
$$n = \frac{t}{T_{1/2}}$$
$$2 = \frac{t}{5730\ years}$$
$$2 \times 5730\ years = t$$
$$\boldsymbol{t = 11460\ years}$$

Q18.7 How much of a 1 g sample of pure radioactive substance would left undecayed after four half-lives?

Given Data

$$No.\,of\ half-lives = n = 4$$
$$Orignal\ sample\ = N_o = 1\ \text{g}$$

To Find

$$remaining\ quantity = N = ?$$

Solution

$$Remaining\ quantity = \frac{1}{2^n} \times orignal\ quantity$$
$$N = \frac{1}{2^n} \times N_o$$
$$N = \frac{1}{2^4} \times 1\ \text{g}$$
$$N = \frac{1}{16} \times 1g$$
$$\boldsymbol{N = 0.0625\ g}$$